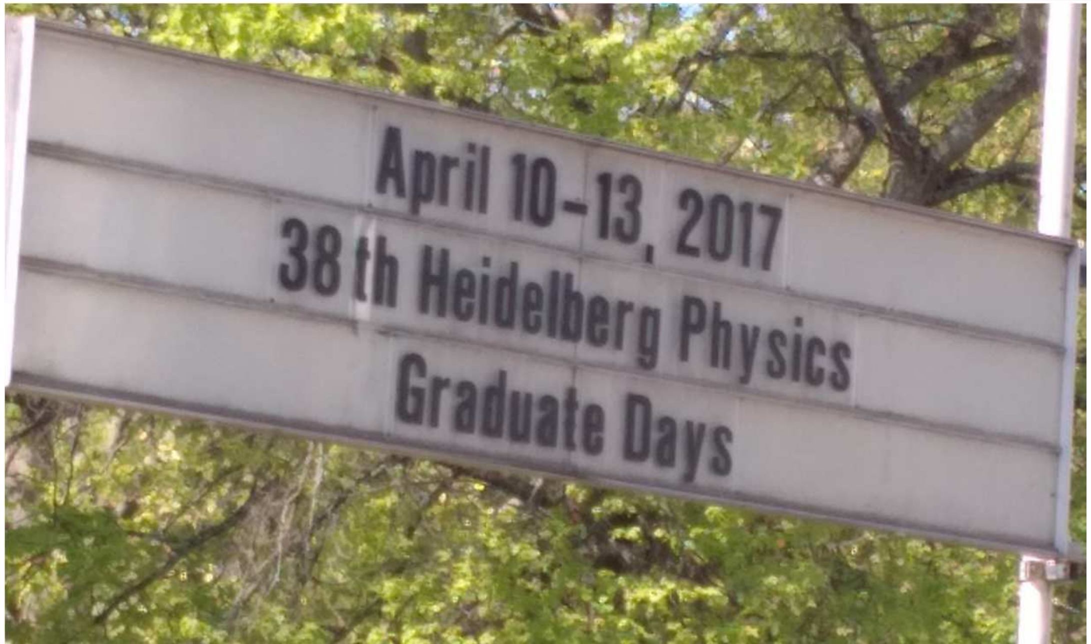
PHYSICS WITH A FEW ATOMS, PHOTONS AND PHONONS

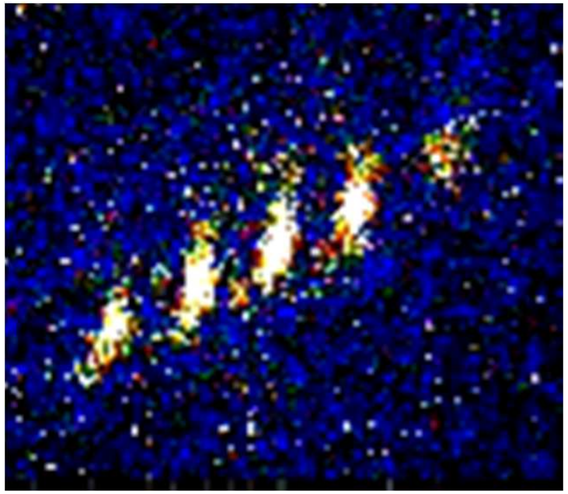


PHYSICS WITH A FEW ATOMS, PHOTONS AND PHONONS

Take home msg

Problems to solve

Assumptions / derivation steps



PHYSICS WITH A FEW ATOMS, PHOTONS AND PHONONS

1. Ion trap – Penning and Paul trap
2. Ion – light interaction
3. Experiments with Penning traps
4. Experiments with Paul traps

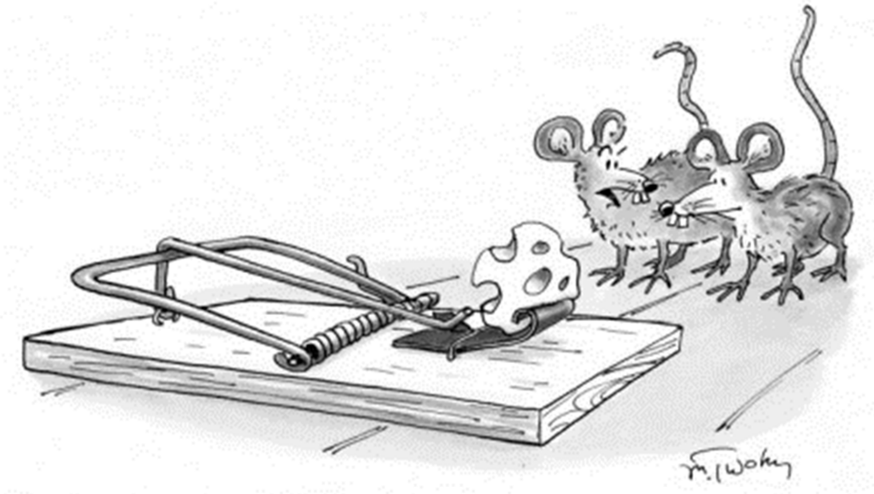
ION TRAP

Content

1. Usefulness
2. Definition and principle of operation
3. Types of ion traps
4. Stability and equation of motion
5. Trap dynamics at low temperature

Definition

- Anything is a trap if you're not careful



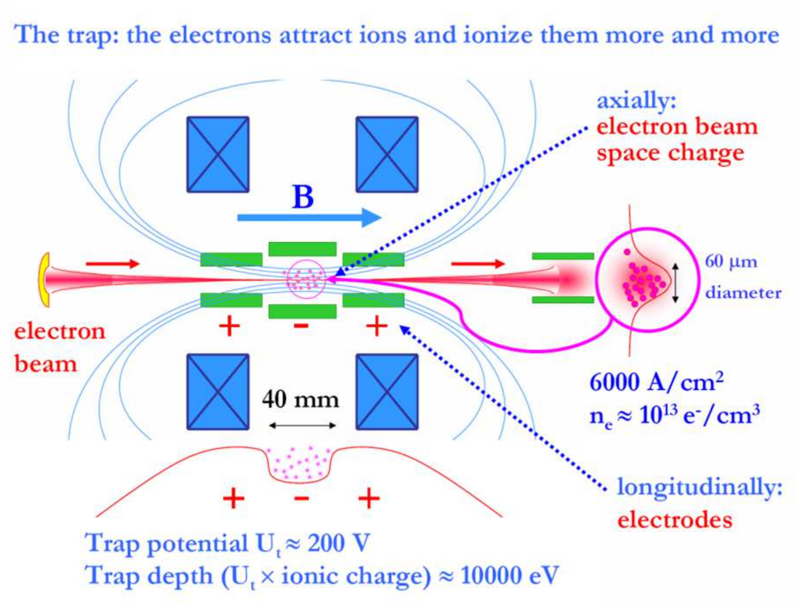
"Careful—it might be a trap!"

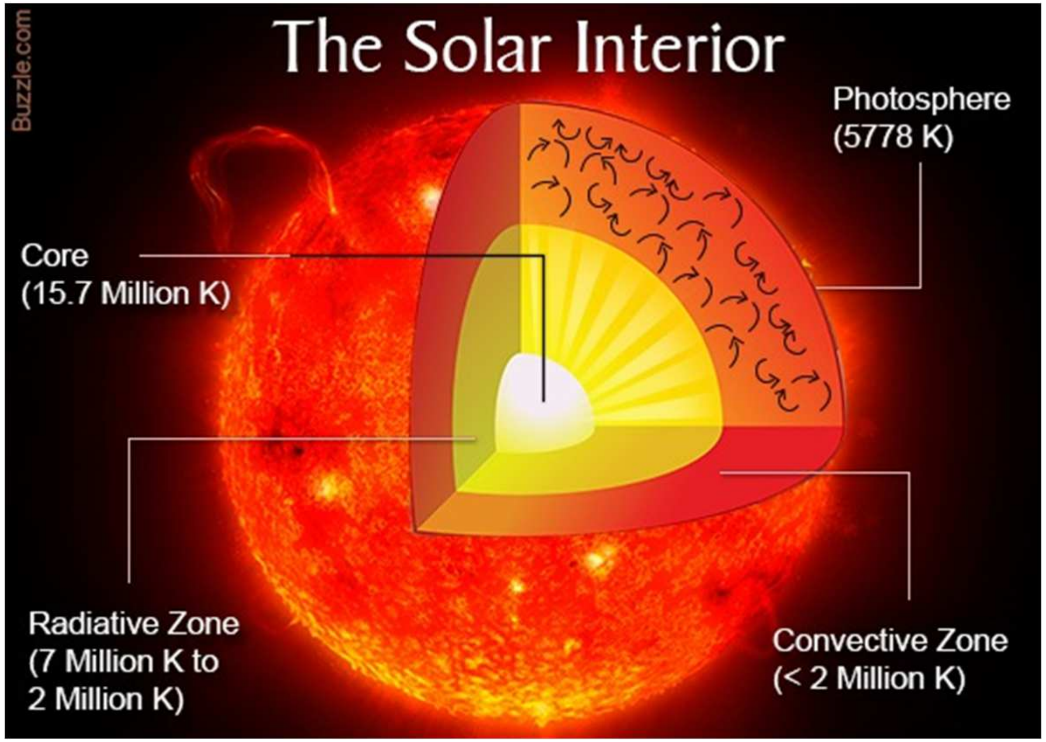
Application

Particle trap

Application	Trap
Lab. Astrophysics	EBIT, super-EBIT etc., Paul trap
Plasma Physics	Penning trap
Condensed matter	Optical lattices, BEC, Paul trap
Biochemistry	FTICR Penning trap
Ultra cold chemistry	Optical lattices, Paul traps
Metrology	Optical lattices, Paul traps
QI & QC	Paul trap: linear and planar
Quantum optics	Paul trap, dipole trap etc.
Nuclear Physics	Paul, Penning, Optical
Particle Physics	Paul trap

Application (Astro. and plasma)





Problem 1a: What is the electron binding energy of hydrogen-like Uranium? Z=92

Application

Particle trap

Application	Trap
Lab. Astrophysics	EBIT, super-EBIT etc., Paul trap
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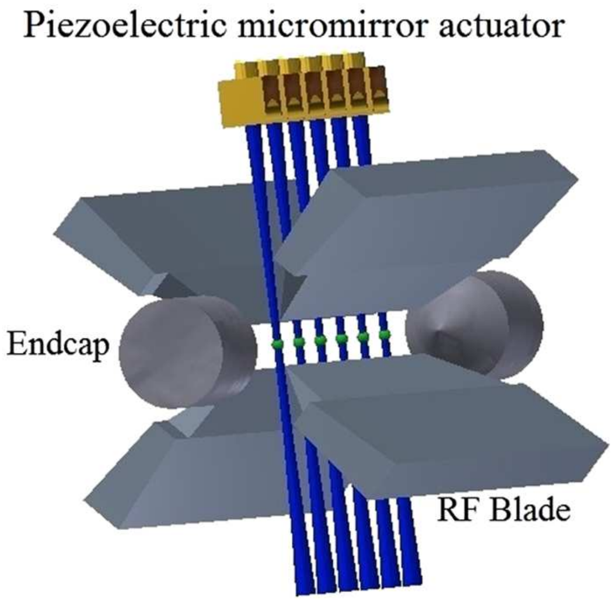
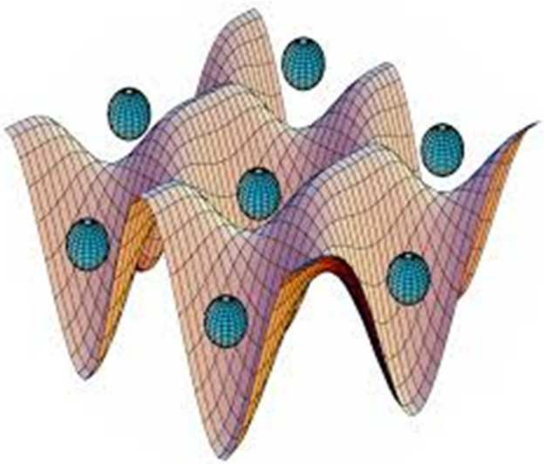
Application (condensed matter)





Frustration in spin system

K. Kim et al. NJP 13, 105003 (2011)

Super fluidity and conductivity

T. Dutta et al. PRL 111, 170406 (2013)

Application

Particle trap

Lab. Astrophysics	EBIT, super-EBIT etc., Paul trap
Plasma Physics	Penning trap
Condensed matter	Optical lattices, BEC, Paul trap
Biochemistry	FTICR Penning trap
Ultra cold chemistry	Optical lattices, Paul traps
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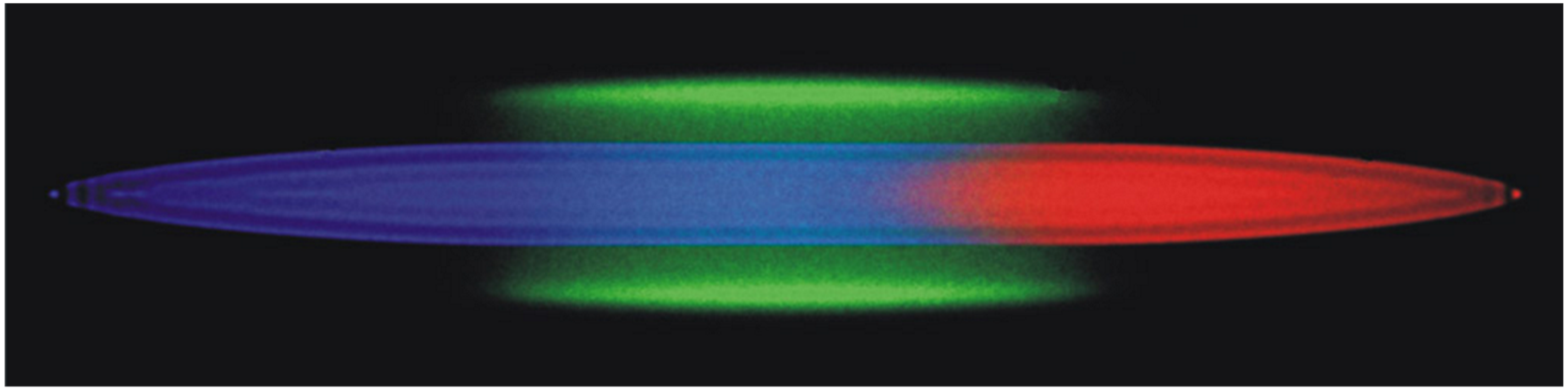
Particle Physics	Paul trap

Application (ultra cold chemistry)



Molecular ions at mK temperature

How was life formed?

Problem 1b: Is this temperature good enough for cold chemistry?

Phys. Rev. Lett. 97, 243005 (2006)

Application

Particle trap

Application	Trap
Lab. Astrophysics	EBIT, super-EBIT etc., Paul trap
Plasma Physics	Penning trap
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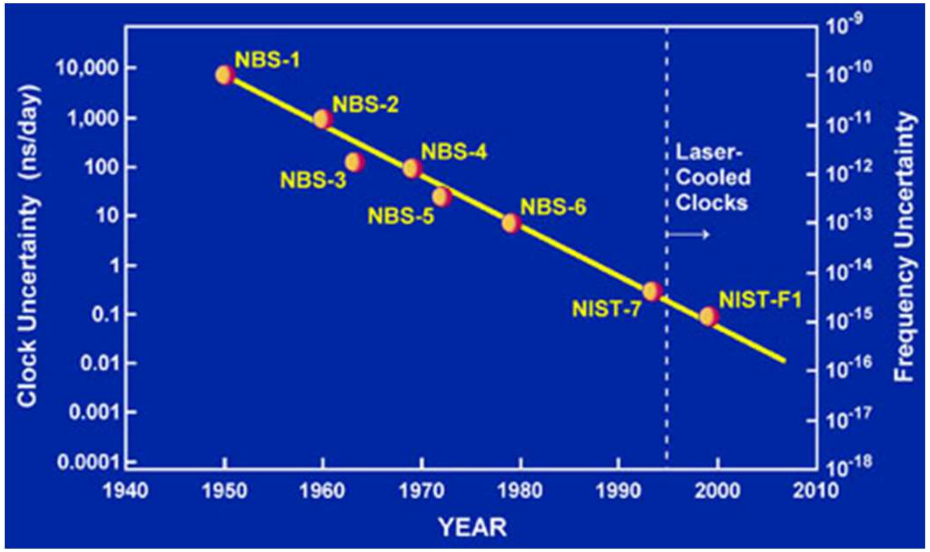
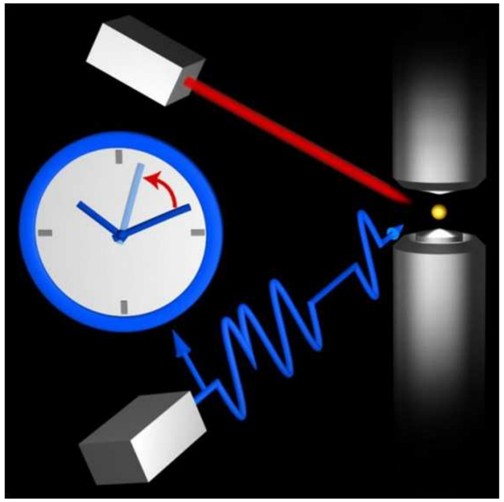
Particle Physics	Paul trap

Application (metrology)

9,192,631,770 Hertz hyperfine transition 133Cs



$$\frac{\delta\nu}{\nu} = 10^{-14}$$



N. Huntemann et al. Phys. Rev. Lett. **116**, 063001 (2016)

$$\frac{\delta\nu}{\nu} = 10^{-18}$$

Problem 1c: with these accuracy how fast or slow will your clock run in day?

Application

Particle trap

Application	Trap
Lab. Astrophysics	EBIT, super-EBIT etc., Paul trap
Plasma Physics	Penning trap
Condensed matter	Optical lattices, BEC, Paul trap
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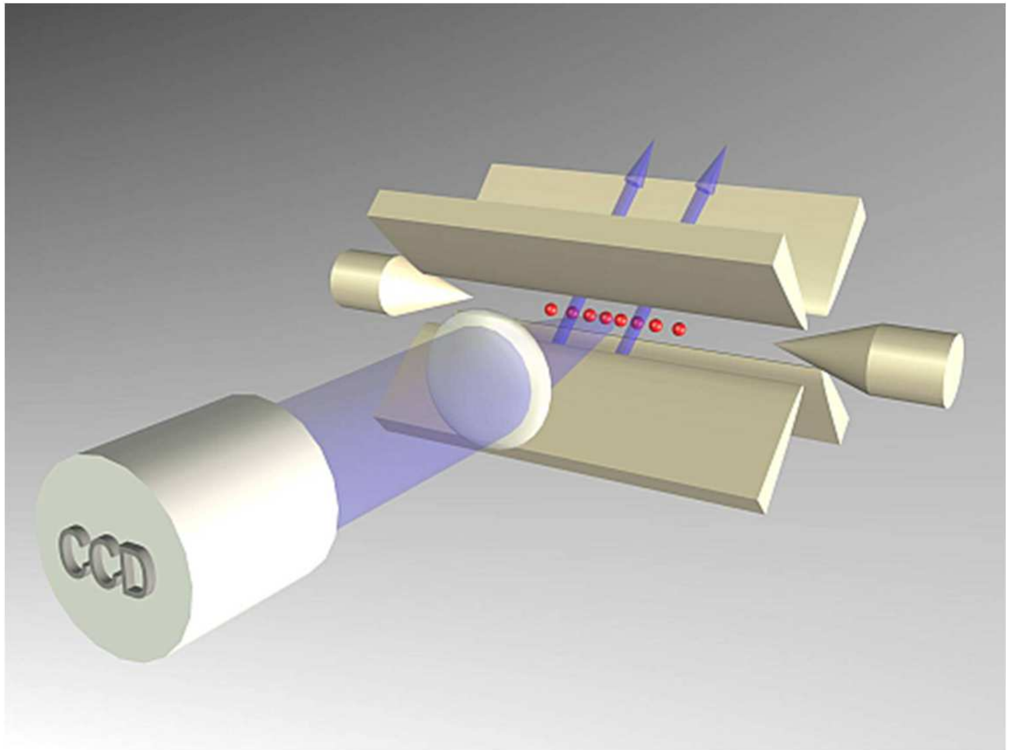
Particle Physics	Paul trap

Application (QC, QO, QI)



IQOQI, University Innsbruck

Application

Particle trap

Lab. Astrophysics	EBIT, super-EBIT etc., Paul trap
Plasma Physics	Penning trap
Condensed matter	Optical lattices, BEC, Paul trap
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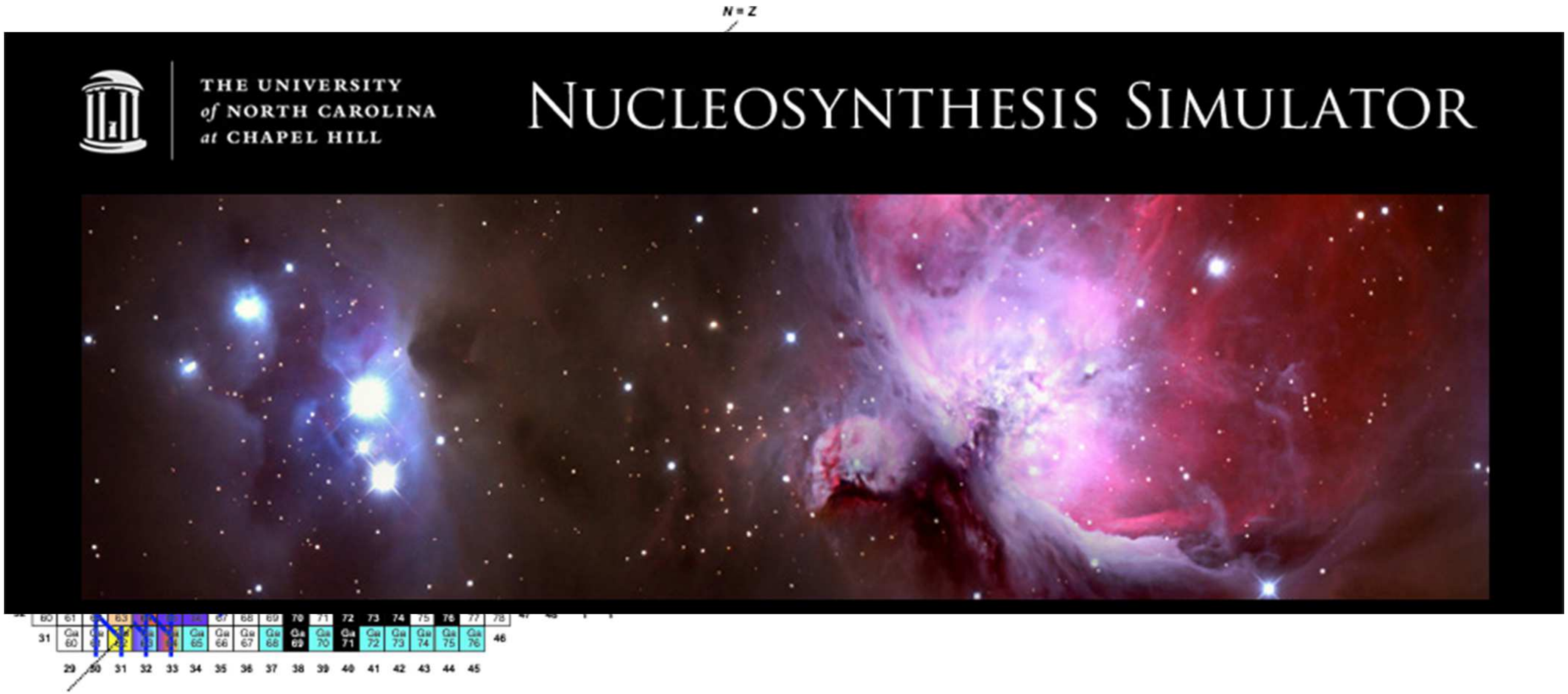
Particle Physics	Paul trap

Application (constituents of the Universe)

Application (Nuclear physics)



Blaum, Klaus et al. Phys.Scripta T152 014017 (2013)

Application

Particle trap

Lab. Astrophysics	EBIT, super-EBIT etc., Paul trap
Plasma Physics	Penning trap
Condensed matter	Optical lattices, BEC, Paul trap
Biochemistry	FTICR Penning trap
Ultra cold chemistry	Optical lattices, Paul traps
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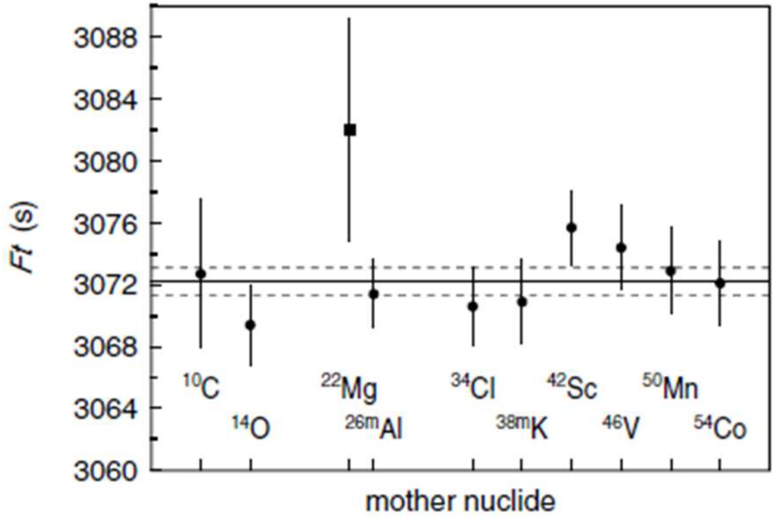
Application (Particle physics)

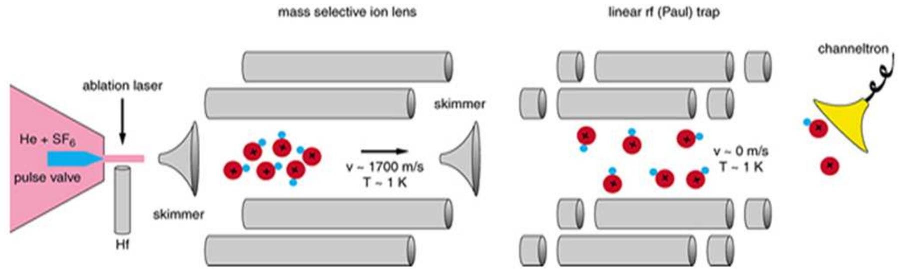
Penning trap mass measurements



Conserved vector current hypothesis
CKM unitarity

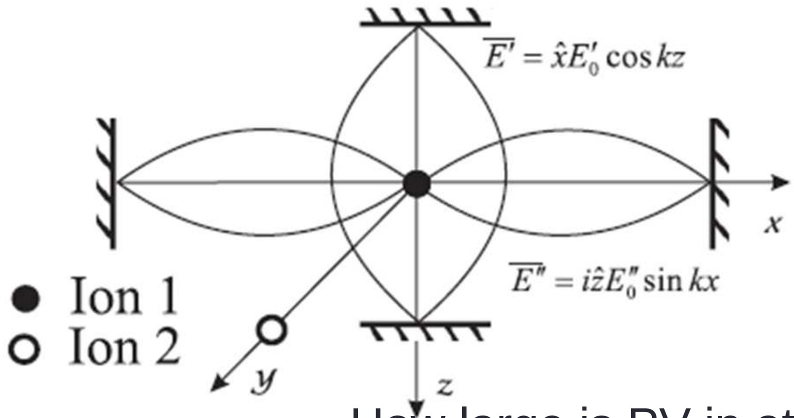
M. Mukherjee et al. PRL 93, 150801(2004)

Paul trap laser spectroscopy



Does electron has a permanent EDM?

EDM ion trap experiment, JILA NIST (E. Cornell)



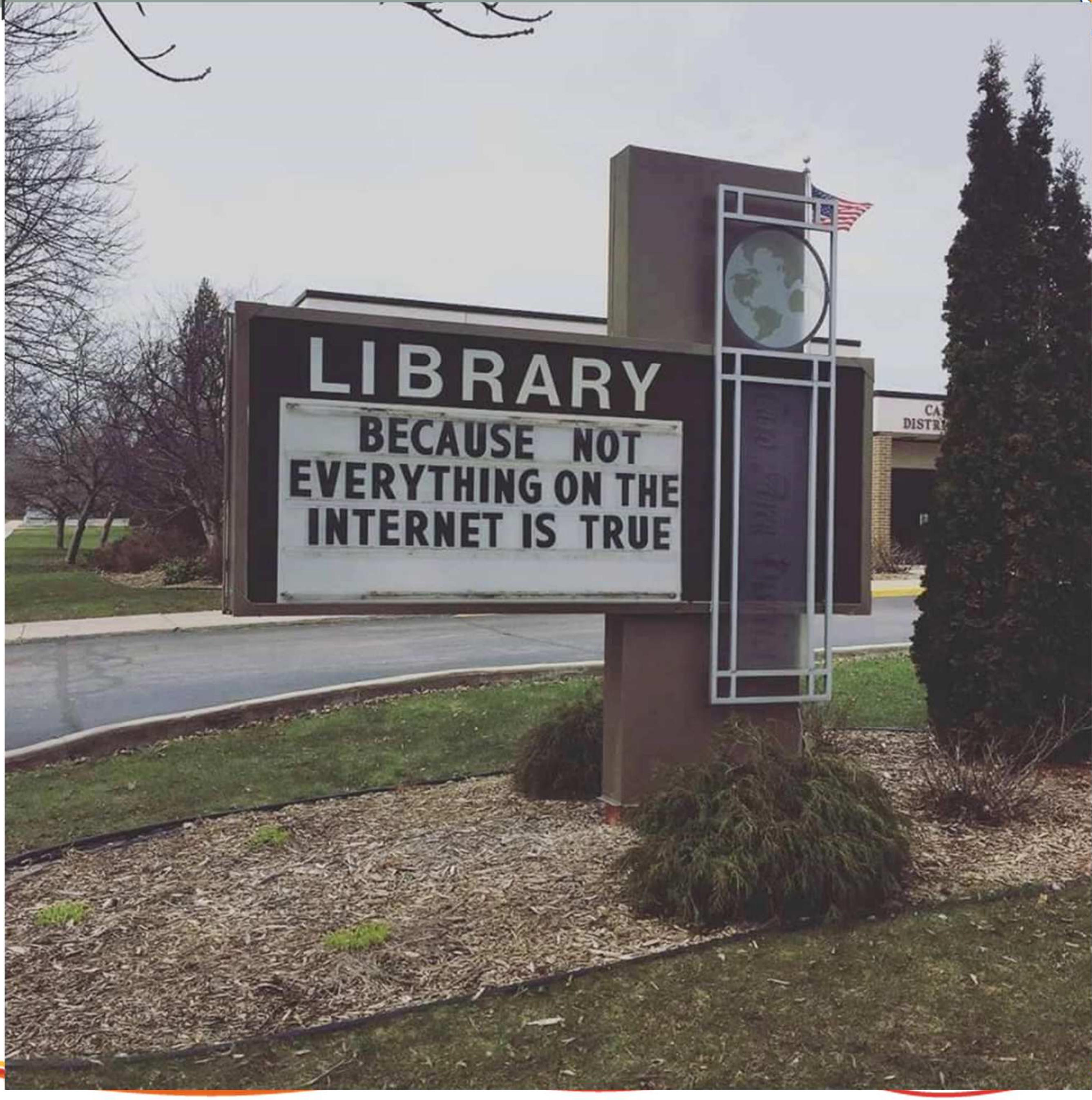
How large is PV in atoms?

CQT/NUS, UW, KVI etc.

Is the SM correct?

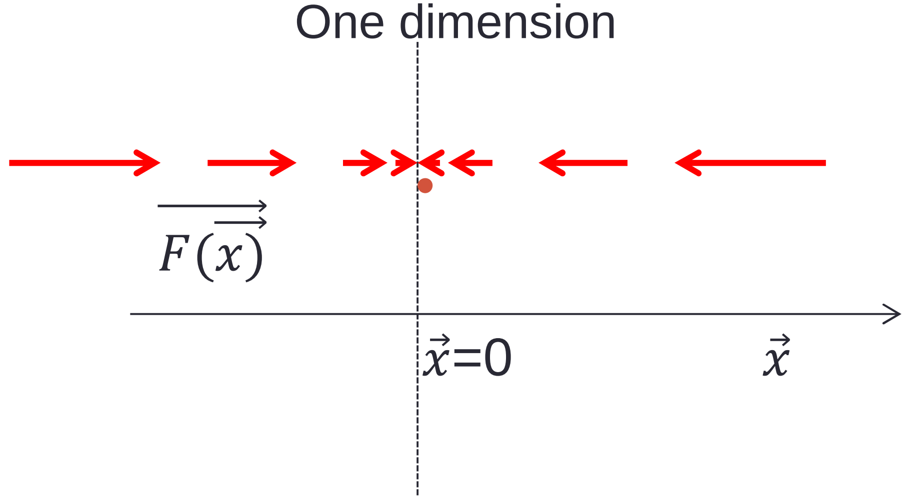
Definition

- Confinement of particle/s in a small volume on space


LIBRARY
BECAUSE NOT
EVERYTHING ON THE
INTERNET IS TRUE

Principle of operation (force argument)

- To confine a particle in one dimension the required force should be proportional to the displacement but opposite in direction



$$\vec{F}(x) = -\alpha\vec{x} \qquad -\frac{dV}{dx} = -\alpha x \qquad V = \frac{\alpha}{2}x^2$$

Trapping charged particle

Continuing to 2D and applying same logic we get

$$V(x,y) = \alpha x^2 + \beta y^2$$

However there is no charge so, *Laplace* condition needs to be fulfilled $\nabla^2 V = 0$

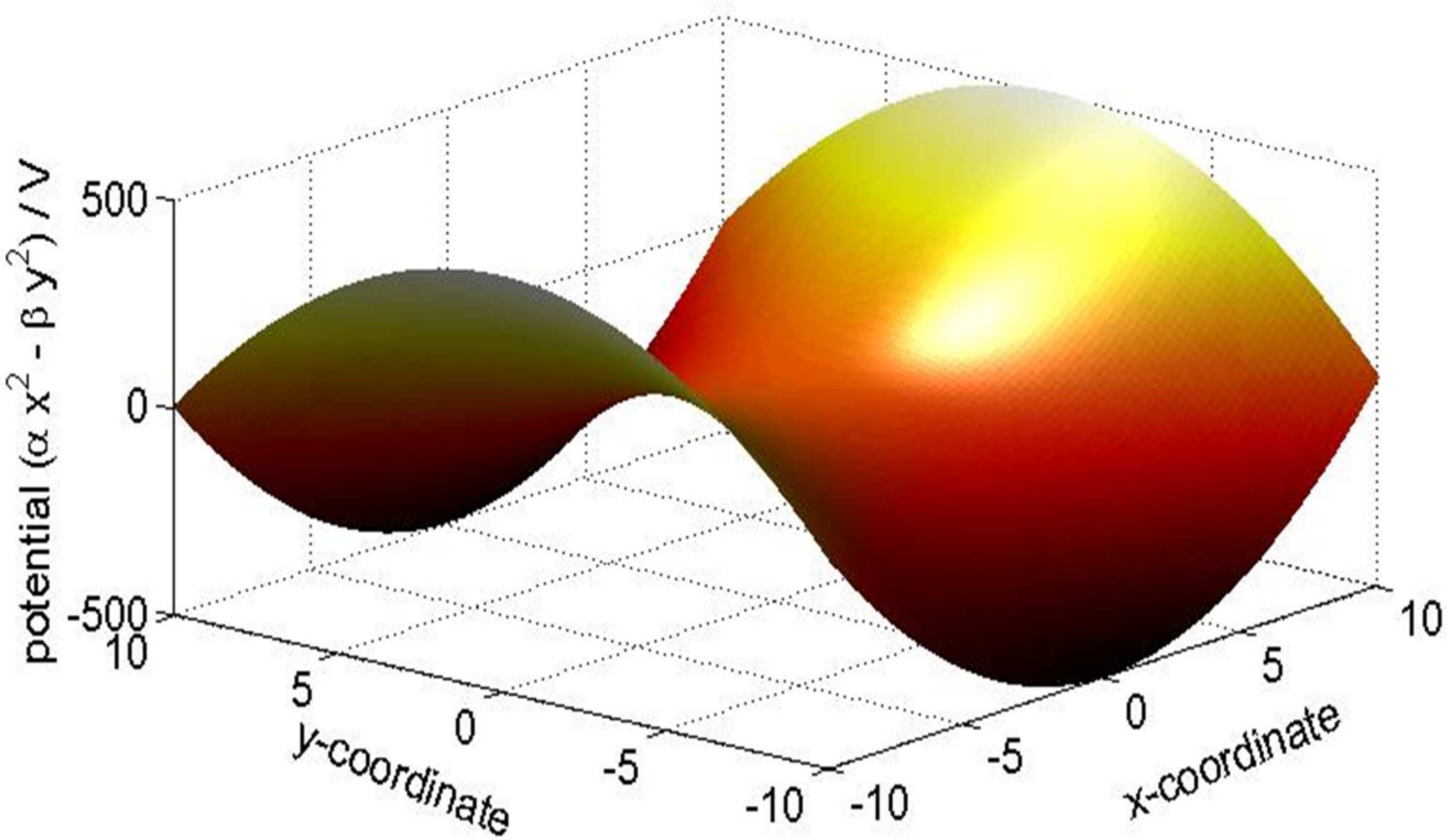
So,

$$V(x,y) = \alpha x^2 - \beta y^2$$

Hence $\vec{F}(x,y) = -\alpha x\hat{x} + \beta y\hat{y}$

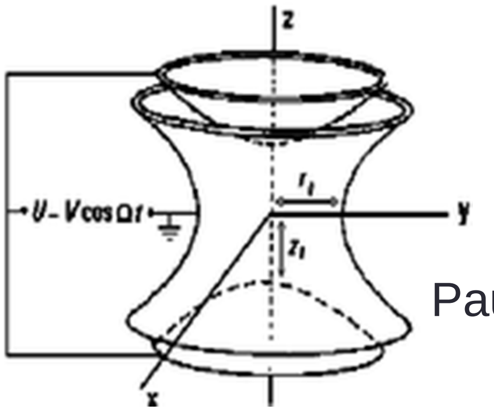
Anti-trapping

Trapping in more than 1D cannot be performed by only static electric potentials



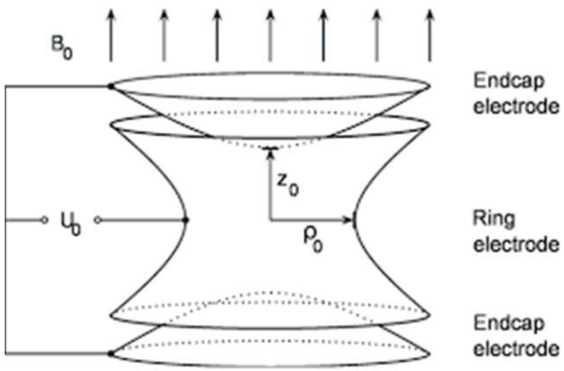
Types of ion traps

Use electro-dynamic field



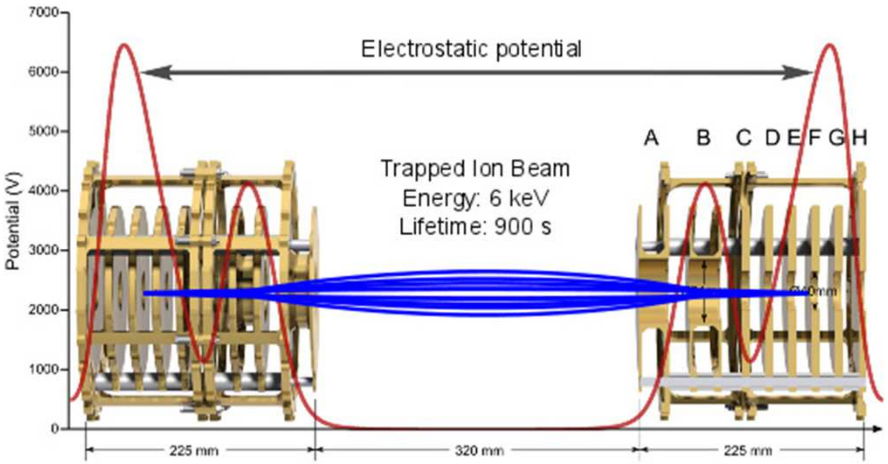
Paul Trap

Use electro-static + magneto-static field



Penning Trap

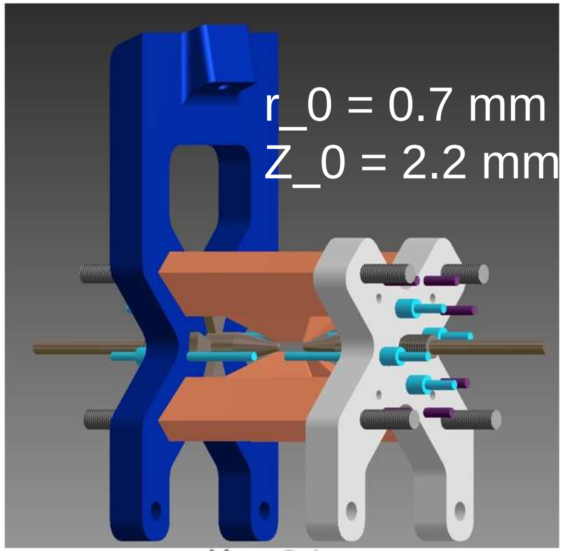
Use electro-static field

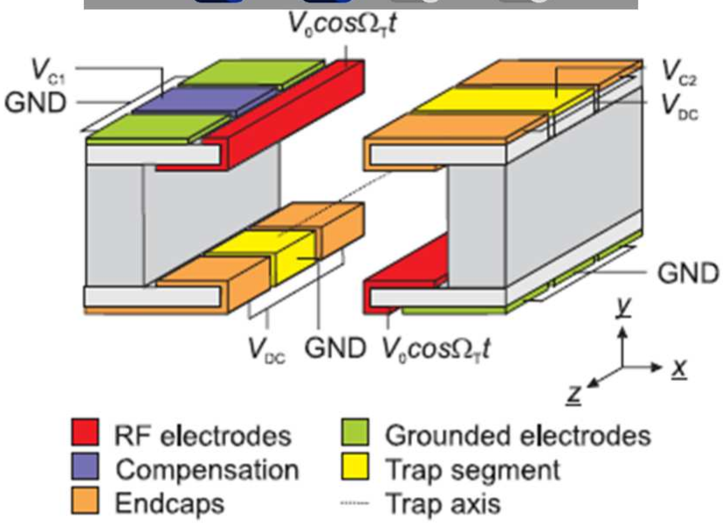


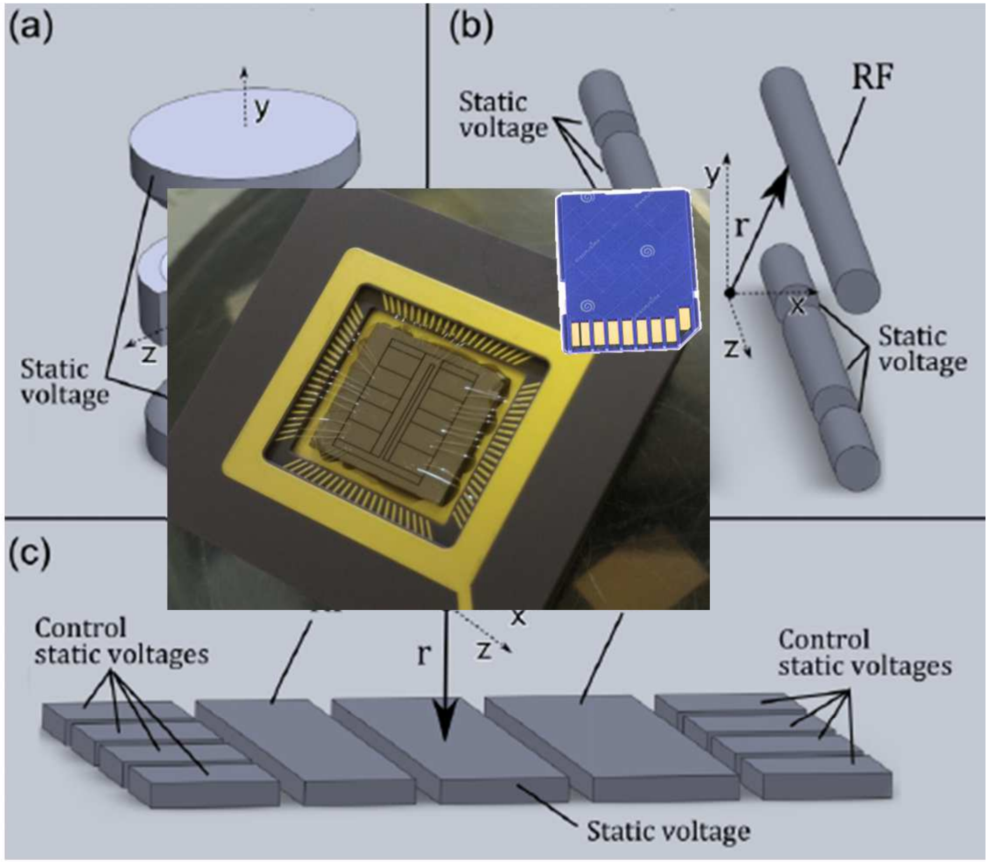
Electro-static Trap

Sources:
Mass Spectrom. Lett. 2015 Vol. 6, No. 4, 112–115 (2015)
https://www.mpi-hd.mpg.de/blaum/high-precision-ms/ptms/basics.en.html

Types of ion traps





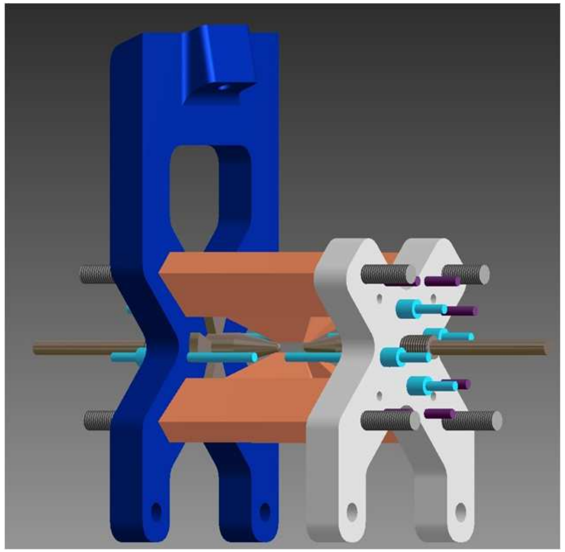


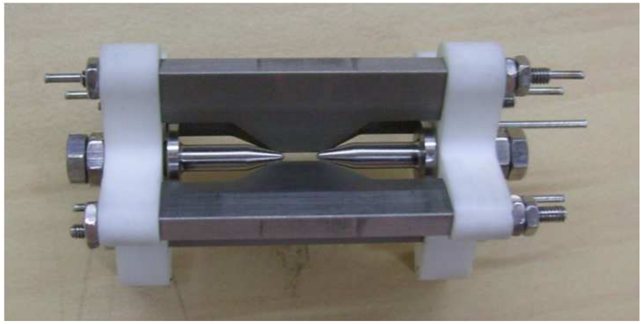
https://arxiv.org/abs/1106.5013

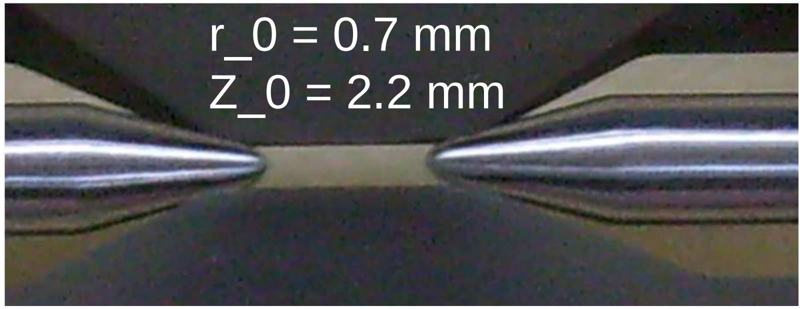
NJP, 8, 232 (2006)

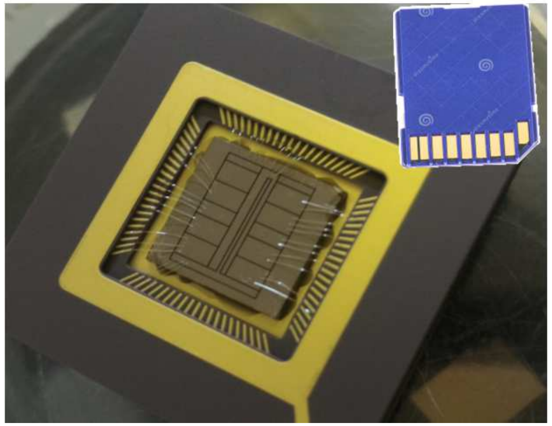
http://coldiongroup.wixsite.com/index

Types of ion traps









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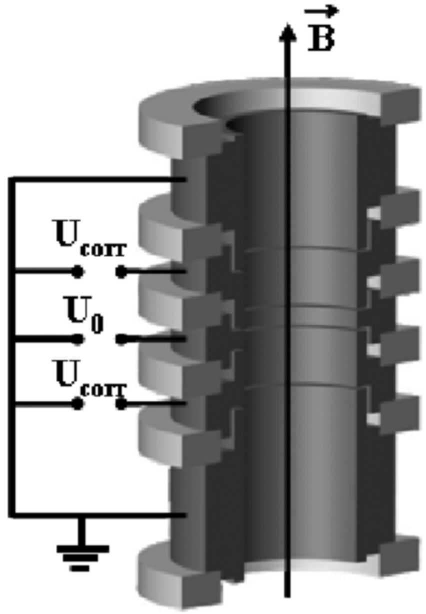
Types of ion traps



Physics world (2013)
MPIK Heidelberg
ISOLTRAP, CERN
Can. J. Physics (2010)

Principles of trap operation ~ History

“We never experiment with just one electron or atom or small molecule. In thought experiments we sometime assume that we do, this invariably entails ridiculous consequences ….In the first place it is fair to state that we are not experimenting with single particle anymore than we can raise Ichthyosauria in the zoo” ------ 1952

“Professor Dr **Wolfgang Paul**, University of Bonn, Federal Republic of Germany and Professor **Hans G. Dehmelt**, University of Washington, USA, have introduced and developed the *ion trap technique* which has made it possible to study a single electron or a single ion with extreme precision.” ------ 1989

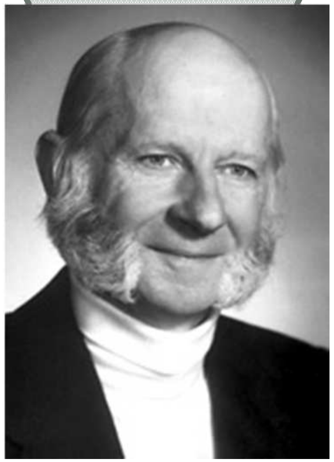
“Serge Haroche and David J. Wineland have independently invented and eveloped methods for measuring and manipulating individual particles while preserving their quantum-mechanical nature, in ways that were previously thought unattainable.” ------ 2012

Principles of trap operation ~ History

Michel Penning

Ion Trap Triology

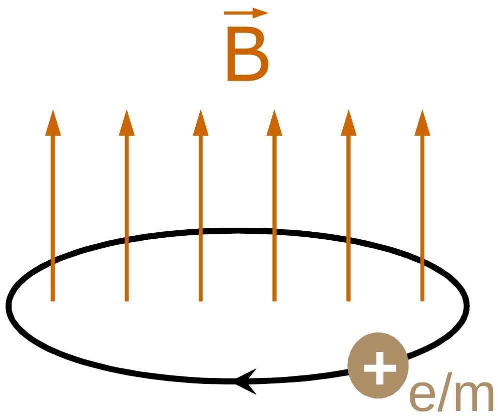
Wolfgang Paul

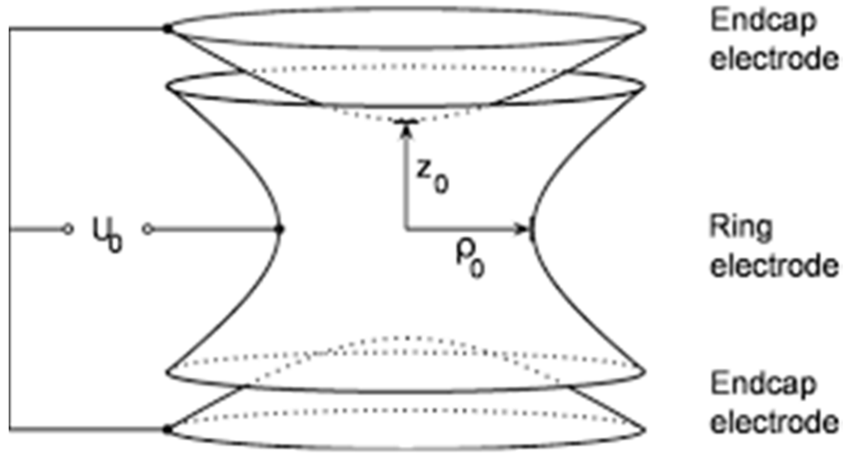


Hans G. Dehmelt

Michael H. Holzscheiter , Physica Scripta. Vol. T59, 69-76, 1995

Penning trap operation





Static magnetic field + Static electric field

Lorentz force:

$$\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$$

Transform to another inertial frame

$$\vec{v} = \vec{v'} + \vec{E} \times \frac{\vec{B}}{B^2}$$

$\vec{E}$	electric field
$\vec{B}$	magnetic field
$\vec{v}$	velocity of charge e

Penning trap operation ~ principle

$$m\frac{d\vec{v}}{dt} = e\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

On substitution

$$m\frac{d\overrightarrow{v'}}{dt} = e(\vec{E} + \left(\overrightarrow{v'} + \vec{E}\times\frac{\vec{B}}{B^2}\right)\times\vec{B}$$

$$\Rightarrow m\frac{d\overrightarrow{v'}}{dt} = e\left(\overrightarrow{v'}\times\vec{B}\right)$$

$$\begin{aligned}\left(\vec{A}\times\vec{B}\right)\times\vec{C} &= -\vec{C}\times\left(\vec{A}\times\vec{B}\right)\\ &= \left(\vec{C}\cdot\vec{A}\right)\vec{B}\\ &- \left(\vec{C}\cdot\vec{B}\right)\vec{A}\end{aligned}$$

$$\vec{E}\cdot\vec{B} = 0$$

With this newly defined velocity, the eq. of motion for a charged particle without electric field is recovered. This means that the role of electric field can be substituted by a $\left(\vec{E}\times\vec{B}\right)$ field.

Therefore balance of force leads to :

Penning trap operation ~ principle

Pure cyclotron motion

$$\frac{mv'^2}{r} = ev'B$$

$$\Rightarrow \omega' = \frac{eB}{m}$$

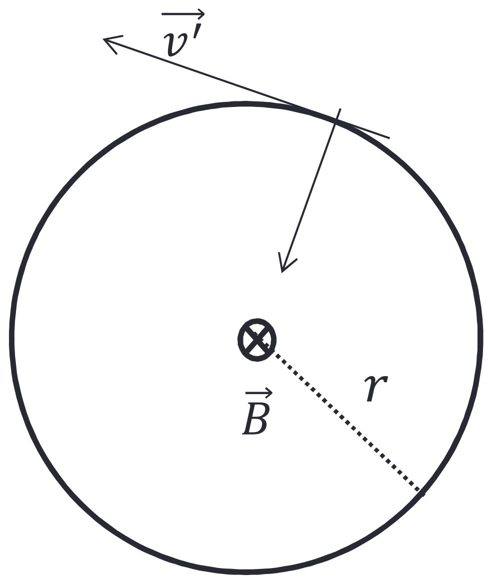
Magnetic field strength

Electric charge

$$\omega' = \frac{eB}{m}$$

Mass

Motion of a charged particle in pure magnetic field

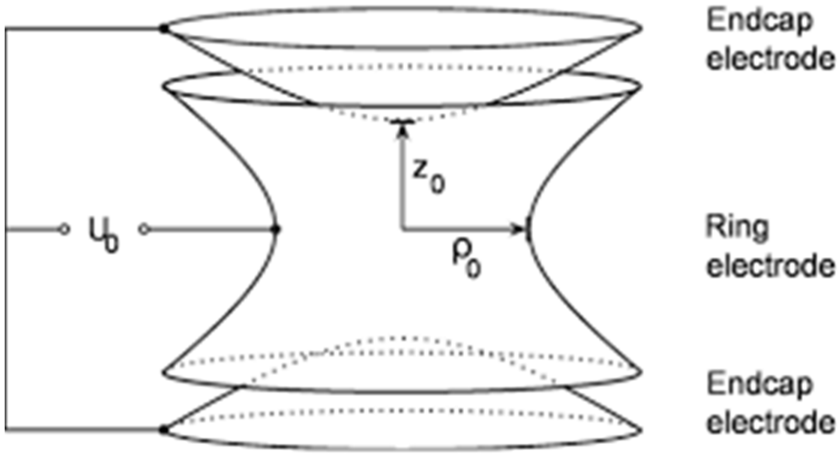


Problem 1: Calculate the cyclotron frequencies of an electron and a singly charged calcium atom in an uniform magnetic field of 1T?
Problem 2: Suppose both electron and positron are confined in a same uniform magnetic field. What would be the difference in their motion?

Penning trap operation ~ equation of motion

The electrostatic part is obtained from the voltages applied to the electrodes which are hyperbolic in shape

$$V(\rho, z) = \frac{\frac{U_0}{2}\left(z^2 - \frac{\rho^2}{2}\right)}{d_0^2}$$



With characteristic dimension

$$d_0^2 = \frac{1}{2}\left(z_0^2 + \frac{\rho_0^2}{2}\right)$$

This type of potential originates by following the surface of the electrodes which forms two hyperbolas namely,

$$z^2 = \left(z_0^2 + \frac{\rho^2}{2}\right)$$

The endcap electrode

$$z^2 = \frac{1}{2}(\rho_0^2 - \rho^2)$$

The ring electrode

Penning trap operation ~ equation of motion

Now combine electro-static and magneto-static parts:

$$m\frac{d\vec{v}}{dt} = e\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\vec{r}(x, y, z) = x\hat{x} + y\,\hat{y} + z\,\hat{z}$$
$$= \rho\,\hat{\rho} + z\,\hat{z}$$

$$\dot{\vec{r}} = \vec{v} = \dot{\rho}\vec{\rho} + \dot{z}\hat{z}$$

since

$$\dot{z}\hat{z} \times \vec{B} = 0$$

The <u>z-motion</u> or axial motion is decoupled/independent from radial/transverse motion denoted by ρ.

$$\ddot{z} + \omega_z^2 z = 0 \quad \ldots(1)$$

$$\omega_z^2 = \frac{eU_0}{md_0^2}$$

The <u>ρ-motion</u> or radial motion is governed by:

$$\ddot{\rho} = \frac{e}{m}\left(\vec{E}_{\vec{\rho}} + \dot{\vec{\rho}} \times \vec{B}\right)$$

Penning trap operation ~ equation of motion

$$\ddot{\rho} = \frac{e}{m}\left(\vec{E}_{\vec{\rho}} + \dot{\vec{\rho}} \times \vec{B}\right)$$

The radial component of the electric field can be obtained as:

$$\overrightarrow{E_{\vec{\rho}}} = -\frac{\partial V}{\partial \rho}\hat{\rho} = \frac{U_0 \vec{\rho}}{2d_0^2} = \frac{1}{2}\frac{m}{e}\ \omega_z^2\ \vec{\rho}$$

The last step is substitution $\omega_z^2 = \frac{eU_0}{md_0^2}$

The equation of motion can now be re-written as :

$$\ddot{\vec{\rho}} - \omega_c\ \dot{\vec{\rho}} \times \hat{B}\ - \frac{1}{2}\omega_z^2\ \vec{\rho} = 0$$ …(2)

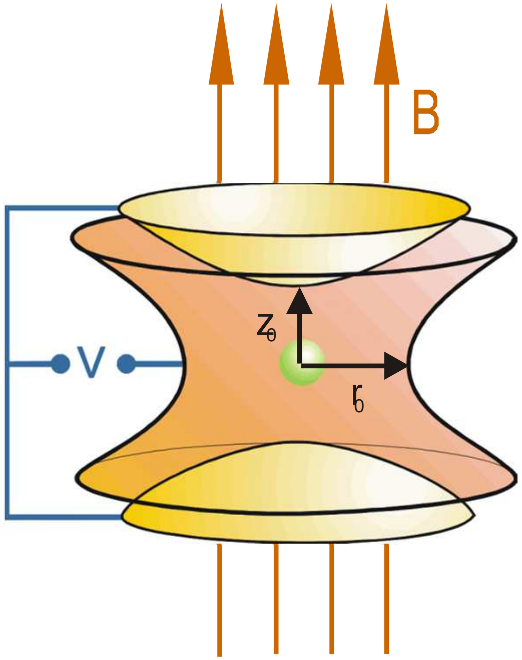
The eigen frequencies obtained by solving eq. (2)

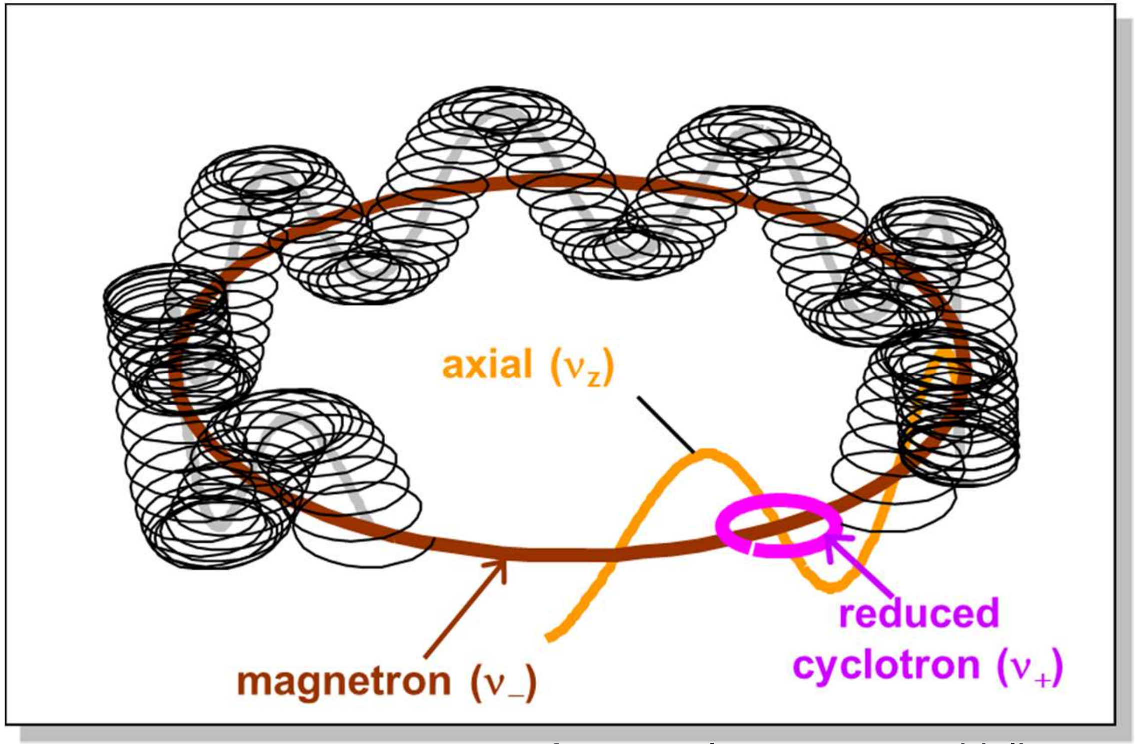
$$\omega_\pm = \frac{1}{2}\left(\omega_c \pm \sqrt{\omega_c^2 - 2\omega_z^2}\right)$$

Modified cyclotron frequency : ω_+

Magnetron frequency : ω_-

Penning trap operation ~ equation of motion





Courtesy: Prof. Dr. K. Blaum, MPIK Heidelberg

Changing to frequency units

$$\omega = 2\pi \times \nu$$

$$\nu_{\pm} = \frac{\nu_c}{2} \pm \sqrt{\frac{\nu_c^2}{4} - \frac{\nu_z^2}{2}}$$

Invariance Theorem

$$\nu_c = \nu_+ + \nu_-$$

Problem 3: Calculate the modified cyclotron and magnetron frequency of a trapped calcium singly charge ion for 1T magnetic field and a trap of dimensions $r_0 = 1$ mm and $z_0 = 2$ mm while endcap voltage of 100V applied to it.?

Penning trap operation ~ Quantum treatment

Re-write the radial velocity components in terms of new radial velocities as

$$\overrightarrow{V^{\pm}} = \vec{\dot{\rho}} - \omega_{\mp}\, \hat{z} \times \vec{\rho}$$

Taking temporal derivative and substituting $\vec{\ddot{\rho}}$

$$\overrightarrow{\dot{V}^{\pm}} = \omega_{\pm} \hat{z} \times \overrightarrow{V^{\pm}}$$ 2 de-coupled motion in the radial plane

Now, consider a gauge for the vector potential as $\vec{A} = \frac{1}{2}\vec{B} \times \vec{\rho}$, the canonical momentum becomes

$$\vec{p} = m\vec{\dot{\rho}} + (e)\vec{A}$$

$$\vec{p} = \frac{m}{2}\left(\overrightarrow{V^{+}} + \overrightarrow{V^{-}}\right)$$

$$\omega_{+} + \omega_{-} = \omega_{c} = \frac{eB}{m}$$

Penning trap operation ~ Quantum treatment

The commutation relations between the components of $V^{\pm}$ are to be derived. So we start with $\rho_k s$ and $p_l s$

$[\rho_k, p_l] = i\hbar\delta_{kl} \Rightarrow [\rho_k, \dot{\rho}_l] = \frac{i\hbar}{m}\delta_{kl}$ $\quad$ k, l are x, y components of ρ

$$[\dot{\rho}_x, \dot{\rho}_y] = \frac{ie\hbar}{m^2c}\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right] = -\frac{i\hbar\left|e\vec{B}\right|}{m^2c} = -i\frac{\hbar\omega_c}{m}$$

Therefore we obtain

$[V_k^+, V_l^-] = 0$ and $\mp\frac{m}{\omega_+ - \omega_-}[V_x^{\pm}, V_y^{\pm}] = i\hbar$ ………(4)

The radial Hamiltonian

$$H_\rho = \frac{1}{2}m\left(\dot{\rho}^2 - \frac{1}{2}\omega_z^2\rho^2\right) \Rightarrow \frac{1}{2}m\,\frac{\omega_+ {V^+}^2 - \omega_- {V^-}^2}{\omega_+ - \omega_-}$$

Magnetron has –ve energy contribution and hence unstable

Penning trap operation ~ Quantum treatment

$[V_k^+, V_l^-] = 0$ and $\mp \frac{m}{\omega_+ - \omega_-}[V_x^\pm, V_y^\pm] = i\hbar$ ……….(4)

This suggests $V_x^\pm$ and $V_y^\pm$ can be treated as conjugate pair of variables (*p*,*q*).

With proper normalization one obtains

$$a_\pm = \left[\frac{m}{2\hbar(\omega_+ - \omega_-)}\right]^{\frac{1}{2}} \left(V_x^\pm \mp i V_y^\pm\right)$$

$$a_\pm^\dagger = \left[\frac{m}{2\hbar(\omega_+ - \omega_-)}\right]^{\frac{1}{2}} \left(V_x^\pm \pm i V_y^\pm\right)$$

$$[a_\pm, a_\pm^\dagger] = 1$$

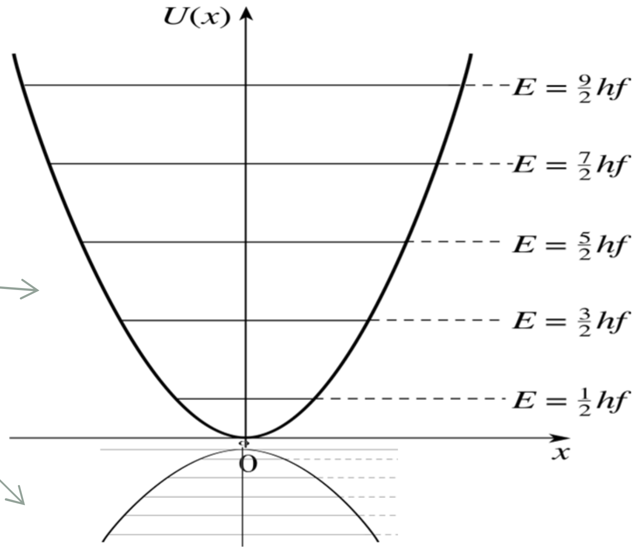
The Hamiltonian is given as:

$$H_\rho = \hbar\omega_+ \left(a_+^\dagger a_+ + \frac{1}{2}\right) - \hbar\omega_- \left(a_-^\dagger a_- + \frac{1}{2}\right)$$

Penning trap operation ~ Quantum treatment

The radial Hamiltonian is given as:

$$H_\rho = \hbar\omega_+\left(a_+^\dagger a_+ + \frac{1}{2}\right) - \hbar\omega_-\left(a_-^\dagger a_- + \frac{1}{2}\right)$$



The axial Hamiltonian is

$$H_z = \frac{P_z^2}{2m} + \frac{m\omega_z^2 z^2}{2}$$

with $[z, P_z] = i\hbar$

Defining operators:

$$a_z^\dagger = \left[\frac{m\omega_z}{2\hbar}\right]^{\frac{1}{2}} z - i\left[\frac{1}{2m\hbar\omega_z}\right]^{\frac{1}{2}} P_z$$

$$a_z = \left[\frac{m\omega_z}{2\hbar}\right]^{\frac{1}{2}} z + i\left[\frac{1}{2m\hbar\omega_z}\right]^{\frac{1}{2}} P_z$$

The axial Hamiltonian is:

$$H_z = \hbar\omega_z\left(a_z^\dagger a_z + \frac{1}{2}\right)$$

Penning trap operation ~ Quantum treatment

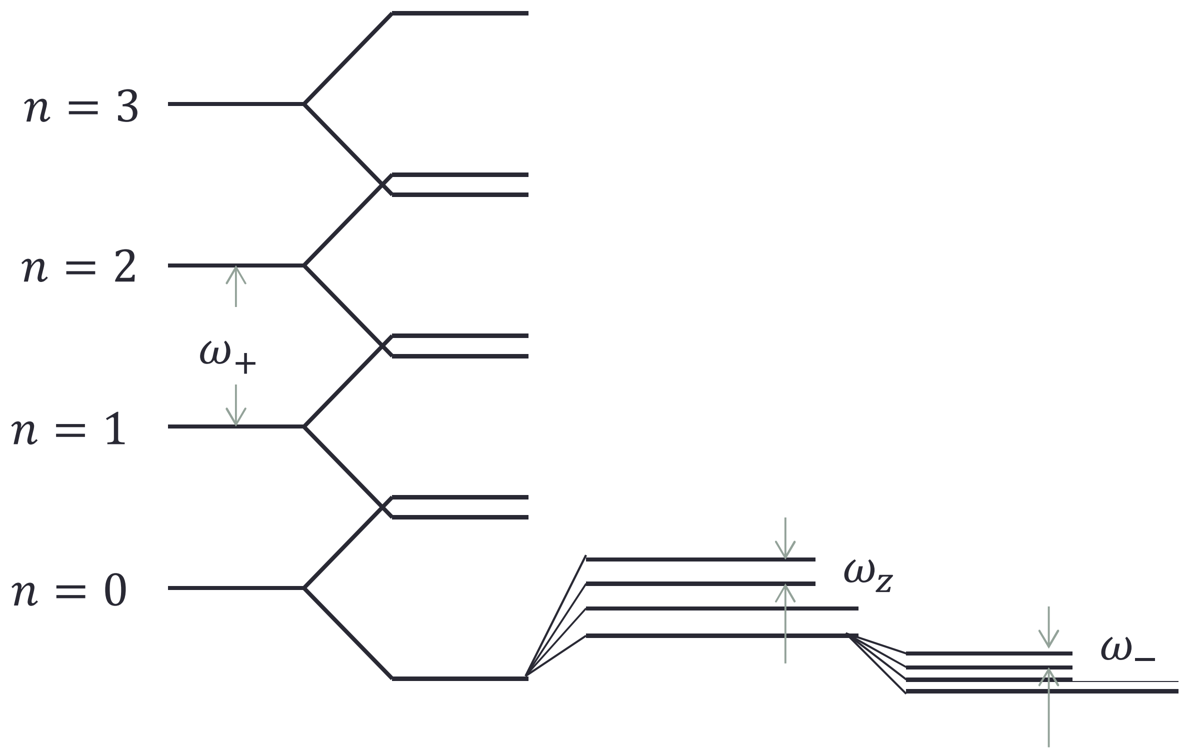
The Hamiltonian is given as:

$$H = \hbar\omega_+ \left(a_+^\dagger a_+ + \frac{1}{2}\right) + \hbar\omega_z \left(a_z^\dagger a_z + \frac{1}{2}\right) - \hbar\omega_- \left(a_-^\dagger a_- + \frac{1}{2}\right)$$

Problem 4: Compare the modified cyclotron, axial and magnetron frequencies of a trapped proton and an electron for 1T magnetic field and a trap of dimensions $r_0 = 1$ mm and $z_0 = 2$ mm while endcap voltage of 100V applied to it ?

Problem 5: Compare the energies in the above three oscillators (for proton and electron) to that of electrode thermal energy kept at 4.2K. Do you need to treat the axial motion quantum mechanically?

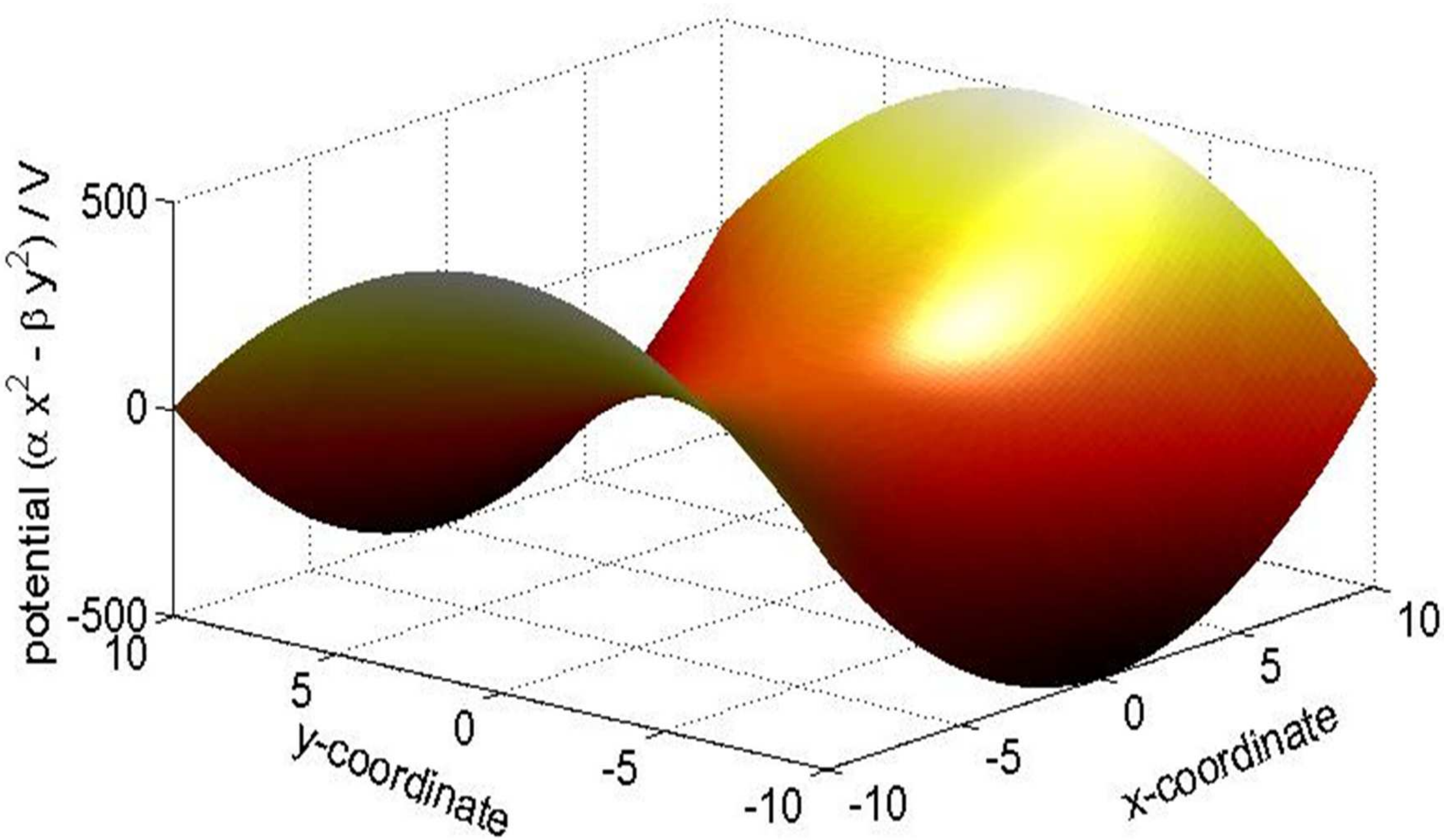
Penning trap operation ~ Quantum treatment



Energy level structure of an single spin-1/2 particle in a Penning trap

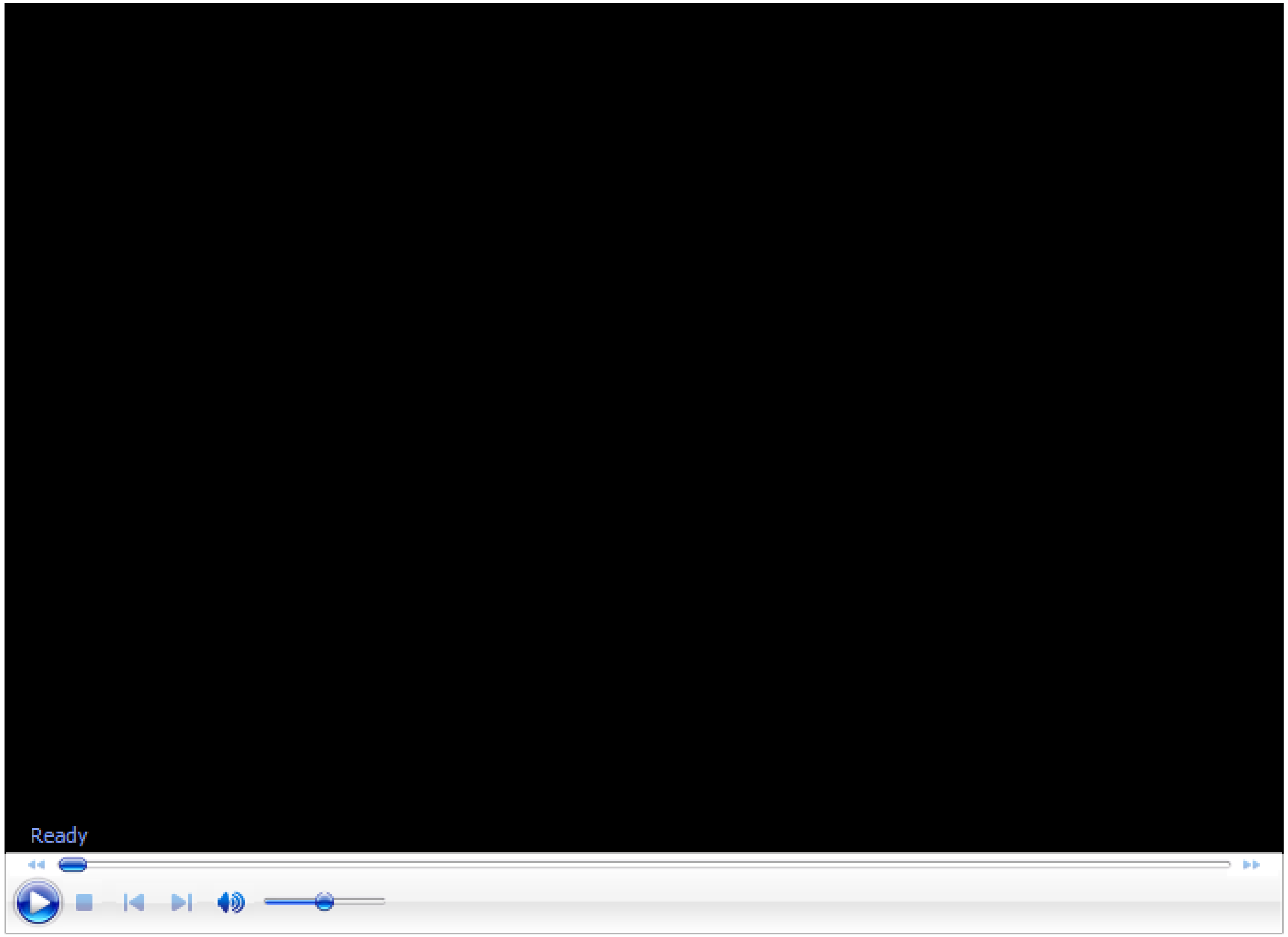
Paul trap operation

Hyperbolic electrode and static potential



Paul trap operation

Hyperbolic electrode and dynamic potential



Paul trap operation

In 3D the potential for confining charged particle needs to be time varying:

$$V(x,y,z) = \frac{U_o}{2}(\alpha x^2 + \beta y^2 + \gamma z^2) + \frac{V_o cos\Omega t}{2}(\alpha' x^2 + \beta' y^2 + \gamma' z^2)$$

In confining region free of source Laplace condition $\nabla^2 V = 0$:

$\alpha + \beta + \gamma = 0$ and $\alpha' + \beta' + \gamma' = 0$

This condition can be achieved in different ways leading to differing geometry of Paul traps

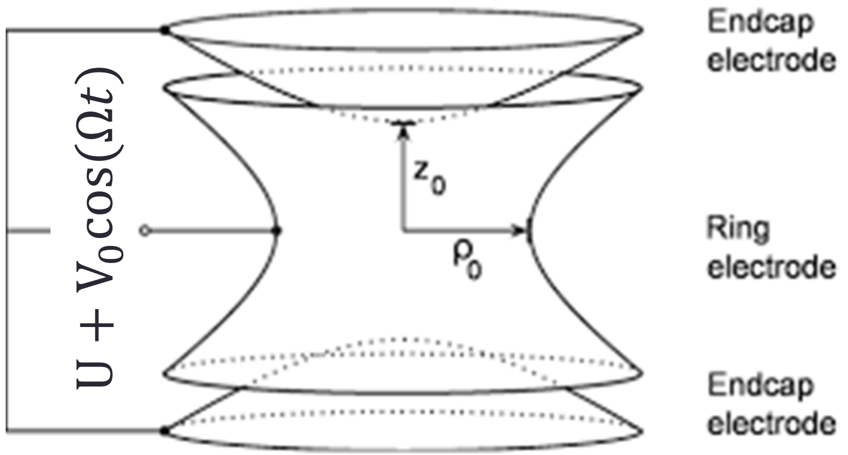
Hyperbolic Paul trap

$\alpha = \beta = \gamma = 0$ and $\alpha' + \beta' = -\gamma'$

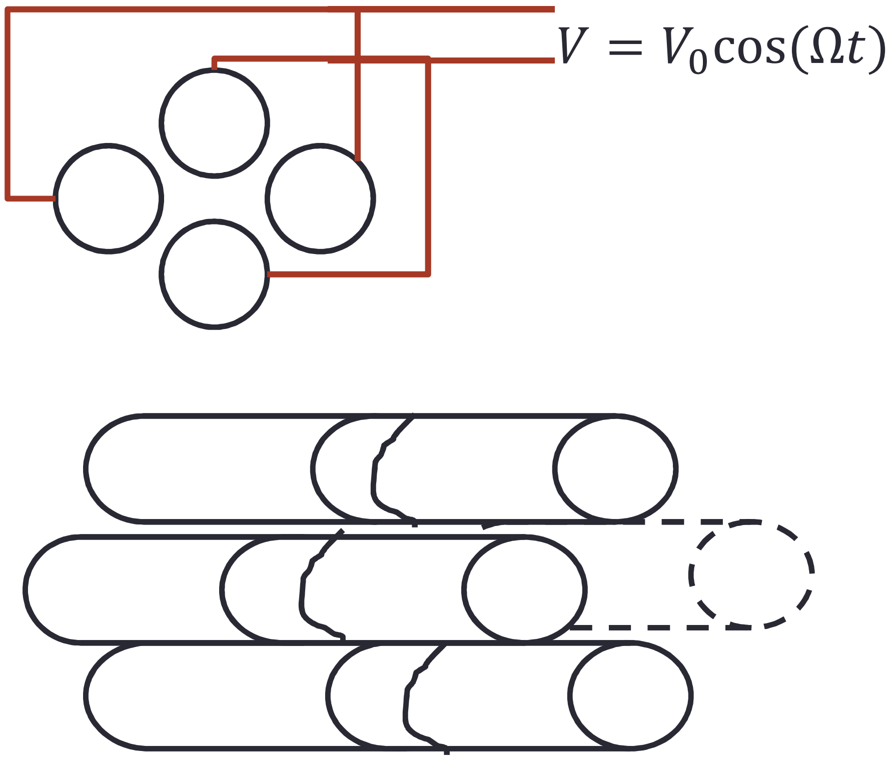
Linear Paul trap

$-(\alpha + \beta) = \gamma > 0$ and $\alpha' = -\beta', \gamma' = 0$

Paul trap operation



Hyperbolic Paul trap



Linear Paul trap

Paul trap operation

Linear traps have the following advantages:

1. Ions are easily accessible by lasers
2. Can form linear chain of ions
3. Individual addressing of ions possible
4. Easy to machine

Paul trap – eq. of motion

$$\ddot{x} = -\frac{Ze}{m}\frac{\partial V}{\partial x} = -\frac{Ze}{m}\left(U_0\alpha + V_0\alpha' cos\Omega t\right)x$$

Substitute

$$\zeta = \frac{\Omega t}{2} \quad a_x = \frac{4ZeU_0\alpha}{m\Omega^2} \quad q_x = -\frac{2ZeV_0\alpha'}{m\Omega^2}$$

$$\boxed{\ddot{x} + (a_x - 2q_x cos2\zeta)x = 0}$$

Mathieu equation

It is second order differential equation similar to simple harmonic oscillator equation <u>but with time dependent frequency term</u>

Paul trap – eq. of motion

Stable solution for the Mathieu equations

$$x(\zeta) = Ae^{\{i\beta_x \zeta\}} \Sigma_{\{n=-\infty\}}^{\infty} C_{2n} e^{\{i2n\zeta\}} + Be^{\{-i\beta_x \zeta\}} \Sigma_{\{n=-\infty\}}^{\infty} C_{2n} e^{\{-i2n\zeta\}}$$

Depends on initial condition

Depends on a_x, q_x not on boundary condition

Using the solution in the equation of motion one obtains the following relation:

$$C_{2n+2} - D_{2n} C_{2n} + C_{2n-2} = 0$$

$$D_{2n} = \frac{[a_x - (2n + \beta_x)^2]}{q_x} \qquad (3)$$

Paul trap – eq. of motion

Using the eq.(3) recursively one obtains:

$$C_{2n+2} = \frac{C_{2n}}{D_{2n} - \frac{1}{D_{2n+2} - \frac{1}{\cdots}}} \qquad C_{2n} = \frac{C_{2n-2}}{D_{2n} - \frac{1}{D_{2n-2} - \frac{1}{\cdots}}}$$

$$\beta_x^2 = a_x - q_x \left(\frac{1}{D_0 - \frac{1}{D_2 - \frac{1}{\cdots}}} + \frac{1}{D_0 - \frac{1}{D_{-2} - \frac{1}{\cdots}}} \right)$$

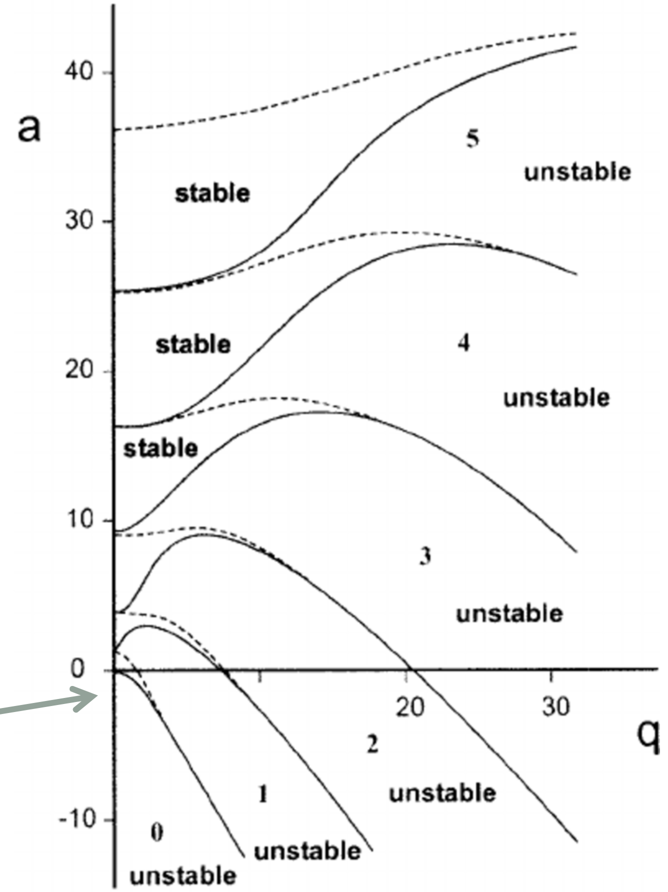
Problem 4: Show that these relations indeed hold

Paul trap – eq. of motion

For stability of ion motion ($x(t \to \infty) < \rho_0$)

$$\beta_{x,y,z} = 0/1$$

Lowest order stability region

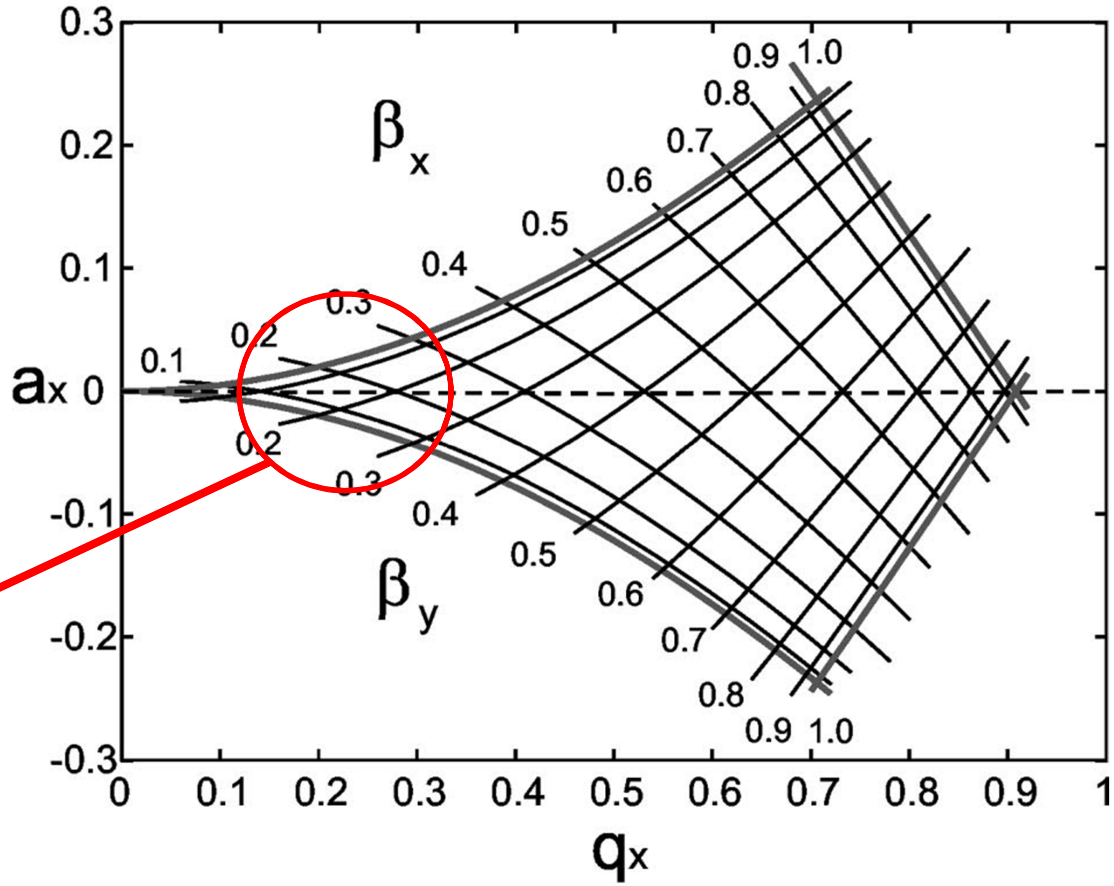


Strutt plot / solutions for 1D Mathieu equation

Paul trap – eq. of motion

Unfortunately life is never that simple, "there are instabilities within the stability region"

Stability region where linear trap works



Working region

$q_x = -q_y$ and $q_z = 0$

Paul trap – eq. of motion

Approximation as per the real linear trap operation:

- Axial confinement is weaker than radial $a_x < q_x$
- Works close to the origin of 1st stability region $|a_x|,\ q_x^2 \ll 1$
- Assuming $C_{\pm 4} \approx 0$

One obtains: $$\beta_x \approx \sqrt{a_x + \frac{q_x^2}{2}}$$

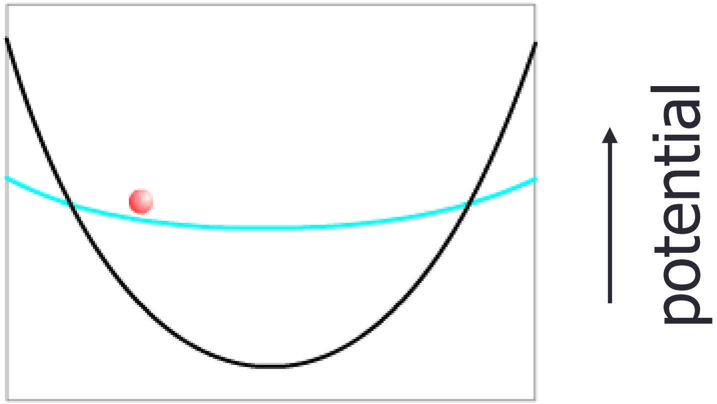
And : $$x(t) \approx 2\mathrm{AC}_0 \cos\left(\frac{\beta_x \Omega}{2} t\right)\left[1 - \frac{q_x}{2}\cos(\Omega t)\right]$$

$$= 2AC_0 \cos\left(\frac{\beta_x \Omega}{2} t\right) - \frac{2AC_0 q_x}{2} \cos\frac{\beta_x \Omega}{2} t \cos \Omega t$$

Secular motion

Micro motion

Paul trap – eq. of motion



$$x(t) = 2AC_0 \cos\left(\frac{\beta_x \Omega}{2} t\right) - \frac{2AC_0 q_x}{2} \cos \frac{\beta_x \Omega}{2} t \cos \Omega t$$

Secular motion

Micro motion

Pseudo-potential approximation

The mean displacement of the ion is negligible within time $\frac{1}{\Omega}$

The total displacement is composed of secular and micro-motion parts

$x = x_s + x_\mu$ reminder $x(t) \approx 2AC_0 \cos\left(\frac{\beta_x \Omega}{2} t\right)\left[1 - \frac{q_x}{2}\cos(\Omega t)\right]$

Secular displacement is large but frequency is slow as compared to micro-motion

$x_s \gg x_\mu$ $\dot{x}_s \ll \dot{x}_\mu$

The time-dependent motion in x is reminder $\ddot{x} + (a_x - 2q_x cos2\zeta)x = 0$

$\ddot{x_\mu} = -(a_x - 2q_x cos2\zeta)\, x_s$ (1) $a_x \ll q_x$

Integrating over time

$x_\mu = -\frac{q_x}{2} cos2\zeta\, x_s$ x_s is constant in one period

Pseudo-potential approximation

Therefore the amplitude of motion is:

$$x = x_s - \frac{q_x}{2} cos2\zeta \, x_s$$

Substituting in $\ddot{x} + (a_x - 2q_x cos2\zeta)x = 0$:

$$\ddot{x} = -(a_x - 2q_x cos2\zeta)\left(1 - \frac{q_x}{2} cos2\zeta\right) x_s$$

$$= -a_x x_s - q_x^2 \cos^2 2\zeta \, x_s + 2q_x \cos 2\zeta \, x_s + \frac{q_x a_x}{2} \cos 2\zeta \, x_s$$

Averaging over one cycle of RF:

$$< \ddot{x}_s >= -\left(a_x + \frac{q_x^2}{2}\right) x_s$$

$$\left\langle \frac{d^2 x_s}{dt^2} \right\rangle = -\left(a_x + \frac{q_x^2}{2}\right) \frac{\Omega^2}{4} x_s$$

Reminder: $\zeta = \frac{\Omega t}{2}$

Pseudo-potential approximation

From pseudo potential model one obtains:

$$\left\langle \frac{d^2 x_s}{dt^2} \right\rangle = -\left(a_x + \frac{q_x^2}{2}\right)\frac{\Omega^2}{4} x_s = -\frac{\beta_x^2 \Omega^2}{4} x_s = -\omega_x^2 \, x_s$$

From solving the Mathieu equation one obtains :

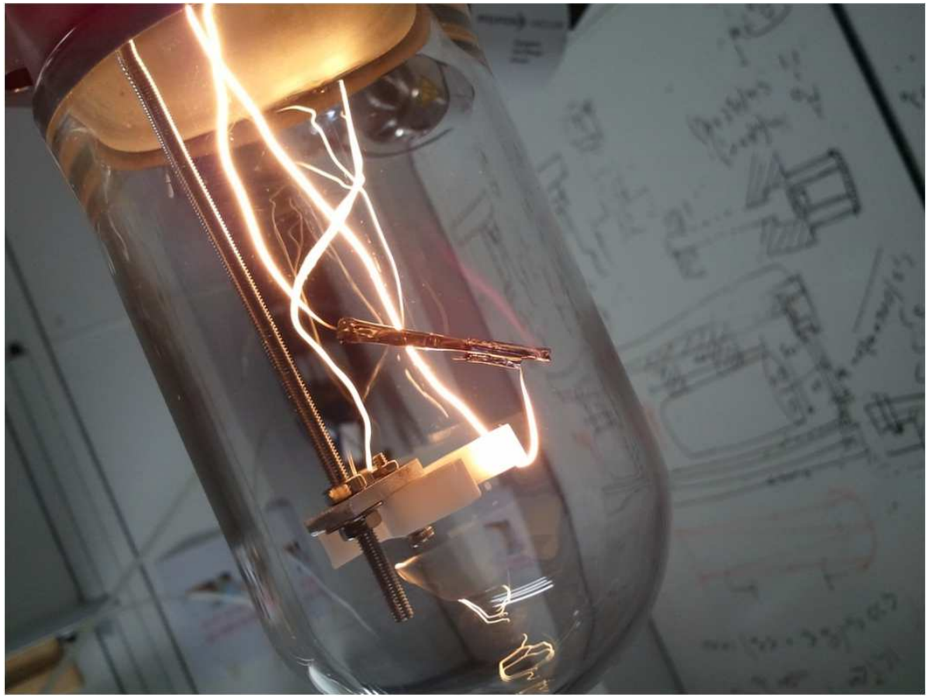
$$x\,(t) = 2AC_0 \cos\left(\frac{\beta_x \Omega}{2} t\right) - \frac{2AC_0 q_x}{2} \cos \frac{\beta_x \Omega}{2} t \cos \Omega t$$

Therefore they match and we observe that the motion is a simple harmonic oscillator motion

Problem 5: Prove that the pseudo-potential trap depth is $\bar{D}_x = \frac{eV_0^2}{4mr_0^2\Omega^2}$ *considering* $a_x = 0$.

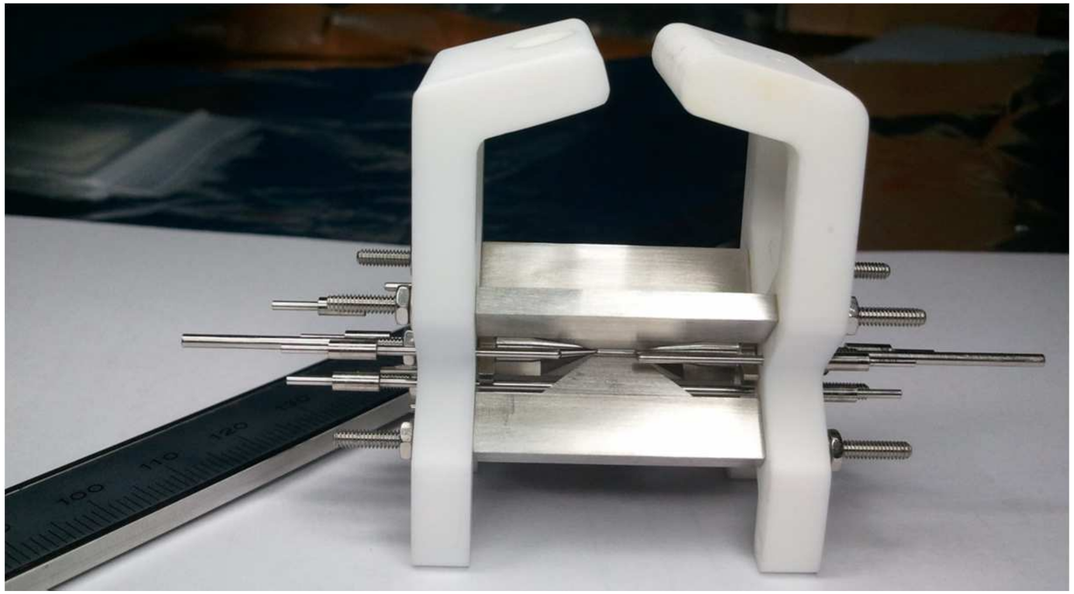
Ion trapping – the steps

Atomic oven – resistive heating



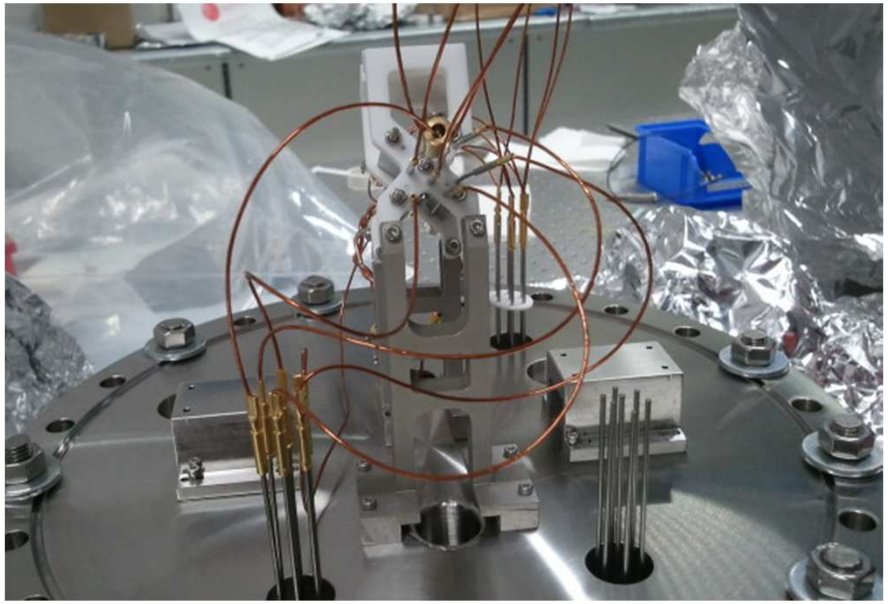
Ion trapping – the steps

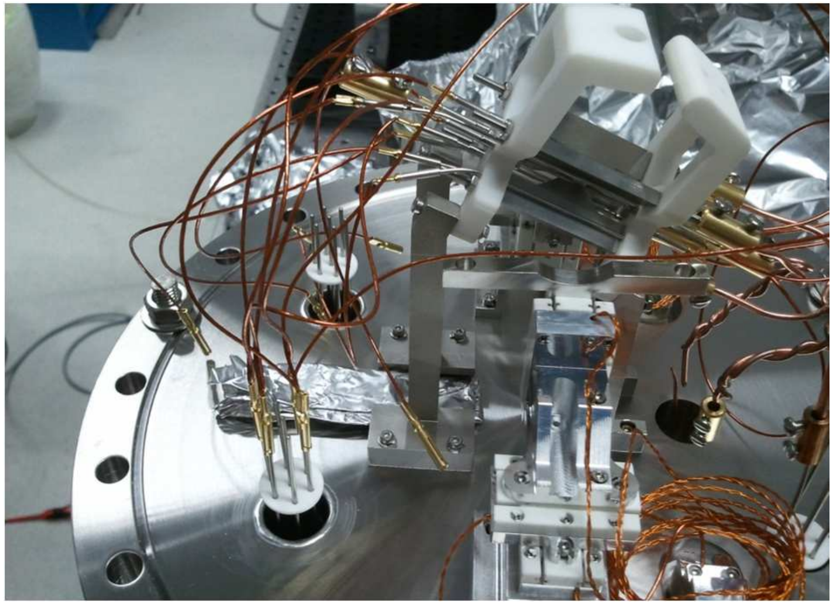
Trap assembly – UHV protocols



Ion trapping – the steps

Trap assembly – UHV protocols



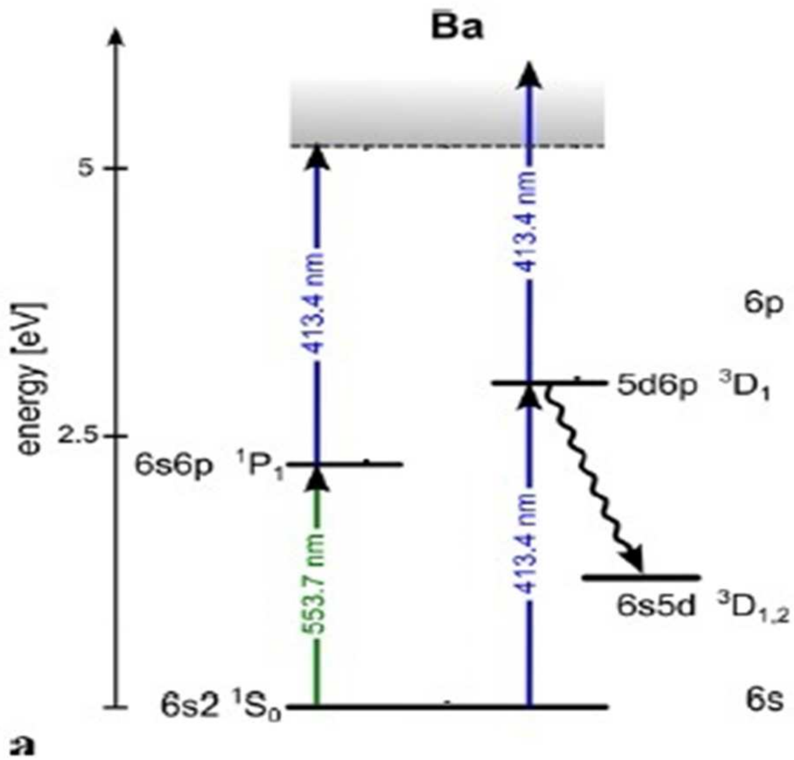


Ion trapping – the steps

Ion creation – in-situ

1. Electron impact
2. Surface ionization
3. Resonant laser ionization
4. etc.

Example for Ba^+

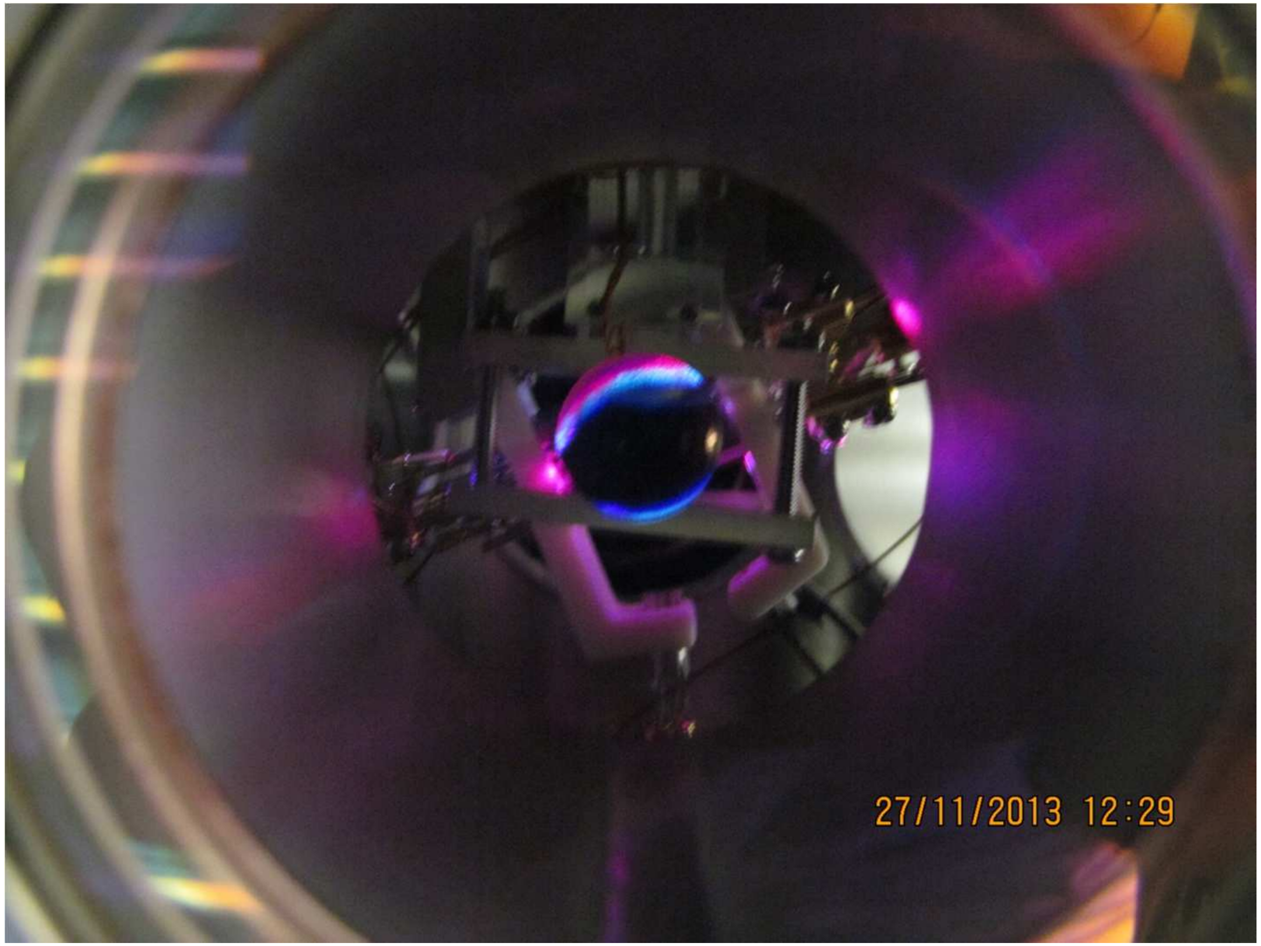


Ion trapping – the steps

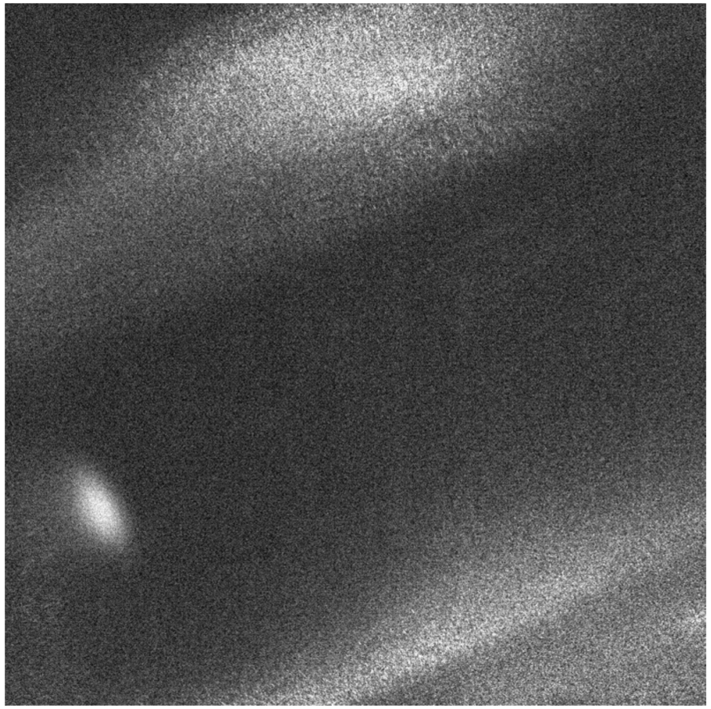
Ion trap drive

Ion trapping – the steps

Ion imaging

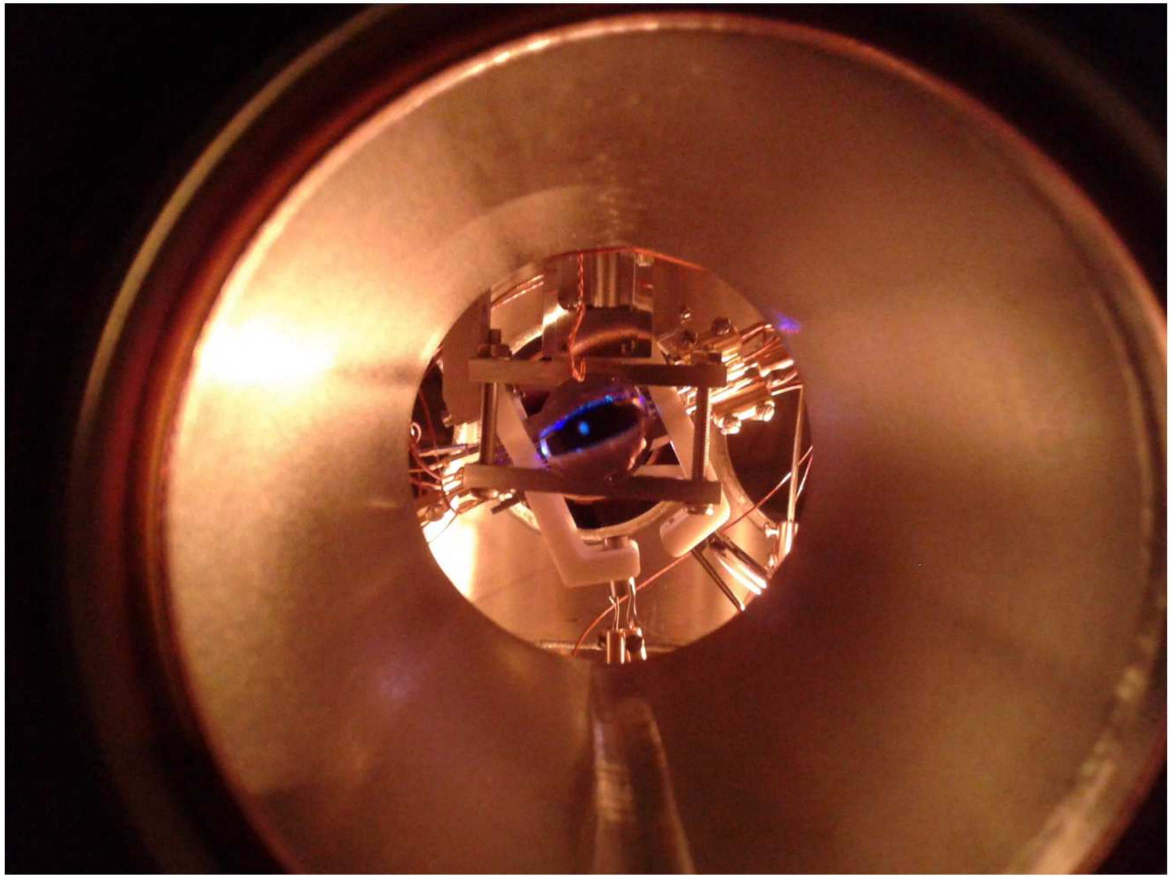


Ion trapping – the steps



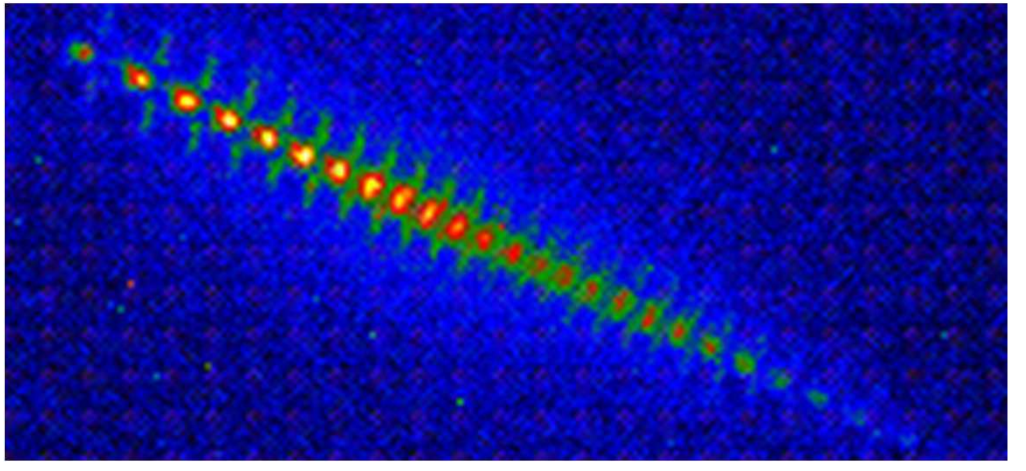
Is there ion?

Ion trapping – the steps



Smart phone image – not Apple!!

Ion trapping – the steps



Chain of 25 ions

CCD image