

PAUL TRAP EXPERIMENTS

Content

1. Quantum information processing
2. Quantum metrology

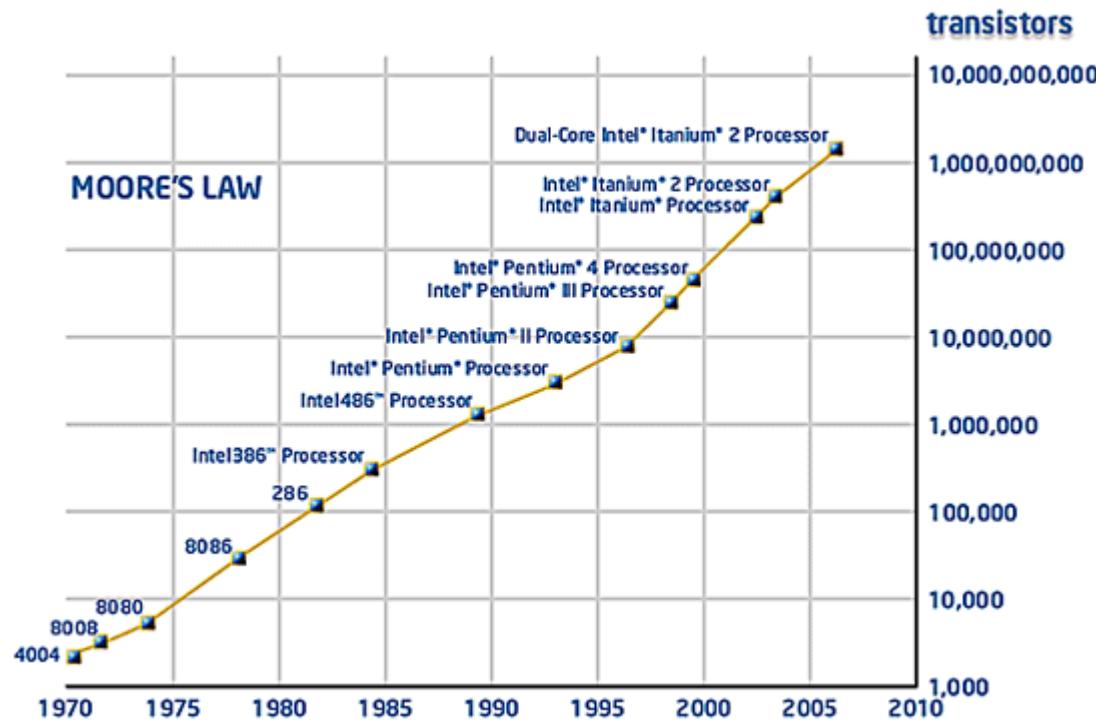
Quantum information processing

During the past forty years astounding advances have been made in the manufacture of computers. The number of atoms needed to represent a bit in memory has been decreasing exponentially since 1950. Likewise the number of transistors per chip, clock speed, and energy dissipated per logical operation have all followed their own improving exponential trends. This rate of improvement cannot be sustained much longer, at the current rate in the year 2020 one bit of information will require only one atom to represent it. The problem is that at that size the behavior of a computer's components will be dominated by the principles of quantum physics. (Williams & Clearwater)

...(T)he first microprocessor only had 22 hundred transistors. We are looking at something a million times that complex in the next generations-a billion transistors. What that gives us in the way of flexibility to design products is phenomenal."

—Gordon E. Moore (1965)

Quantum information processing

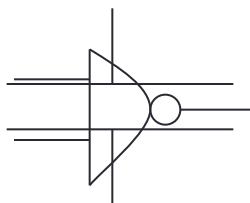


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—Gordon E. Moore (1965)

Quantum information processing

	Classical	Quantum
Memory	BIT	QUBIT
Program	Algorithm	Q-algorithm
Processor	Gates	Q-gates



NAND gate

Quantum information processing

Quantum unit of information: Qubit

Any two level quantum mechanical system **BUT**

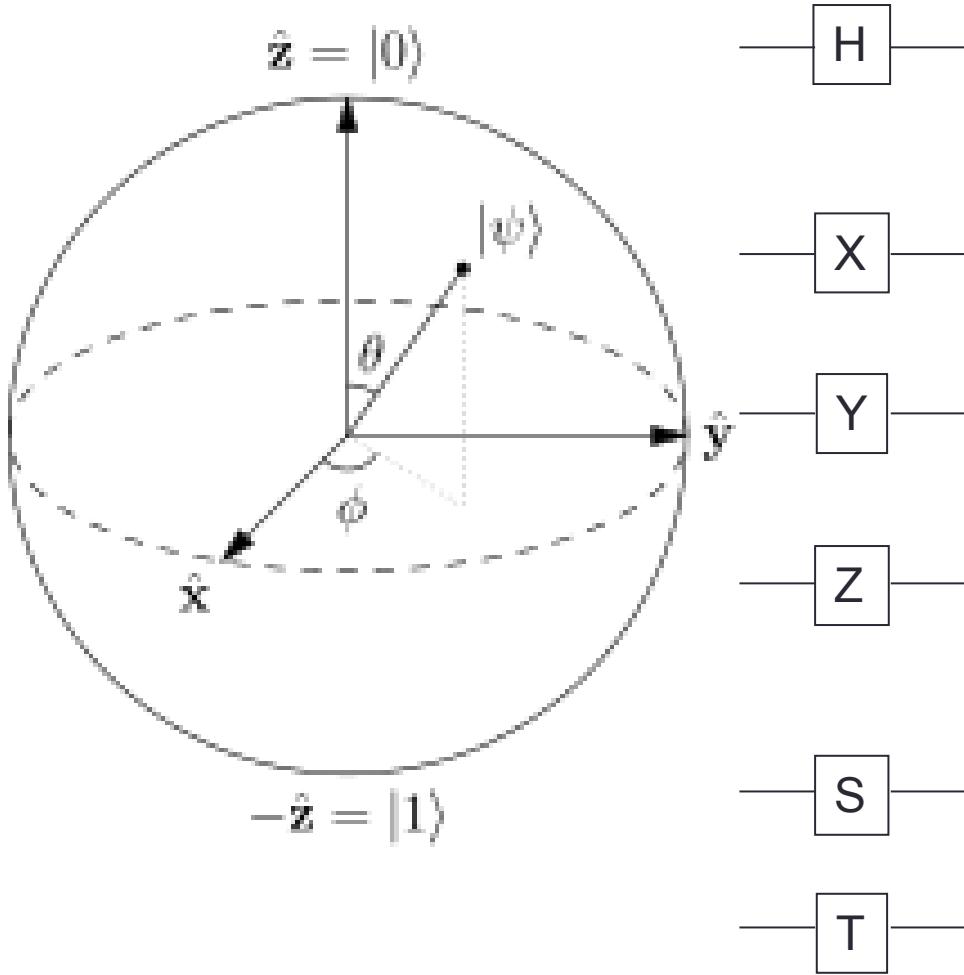
- Possibility to initialize
- Robust against external changes
- Possibility to manipulate using external fields
- Scalable
- Possibility to measure the final state

$$\begin{aligned}|q\rangle &= a|0\rangle + b|1\rangle \\ &= e^{i\gamma} \left(\cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \right)\end{aligned}$$



Quantum information processing

Single qubit gates



Hadamard

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Pauli-X

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Pauli-Y

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

Pauli-Z

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Phase

$$\begin{bmatrix} 1 & 1 \\ 1 & e^{i\phi} \end{bmatrix}$$

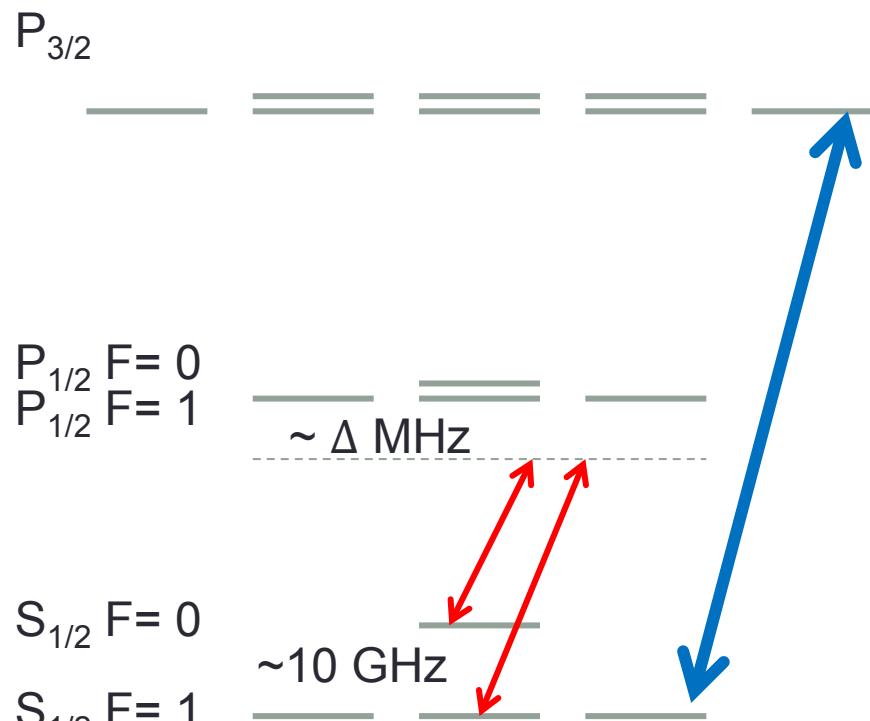
$\Pi/8$

Quantum information processing

Universal 2 qubit gate (C)ontrolled-NOT

Input 2 qubits	Quantum gate	Output 2 qubits
$ q_1\rangle$ $ 00\rangle$ $ q_1\rangle$ $ 01\rangle$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	$ 00\rangle$ $ q_1\rangle$ $ 01\rangle$
$ q_2\rangle$ $ 10\rangle$ $ q_2\rangle$ $ 11\rangle$		$ 11\rangle$ $ q_1\rangle \oplus q_2\rangle$ $ 10\rangle$

Quantum information processing

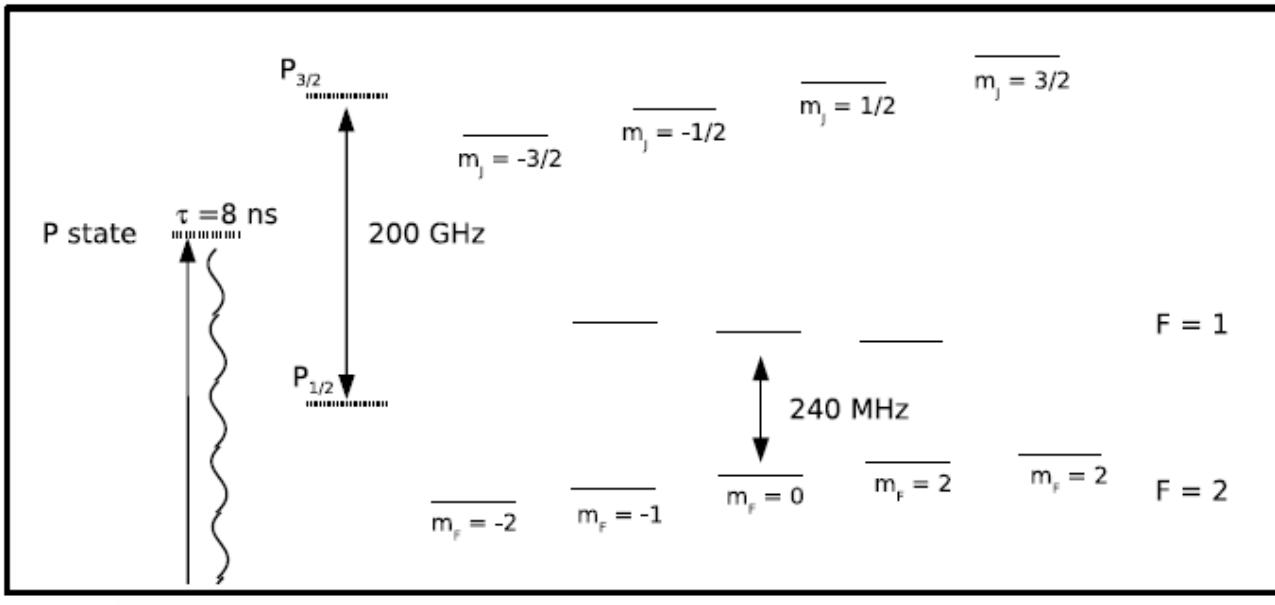


Hyperfine Qubit:

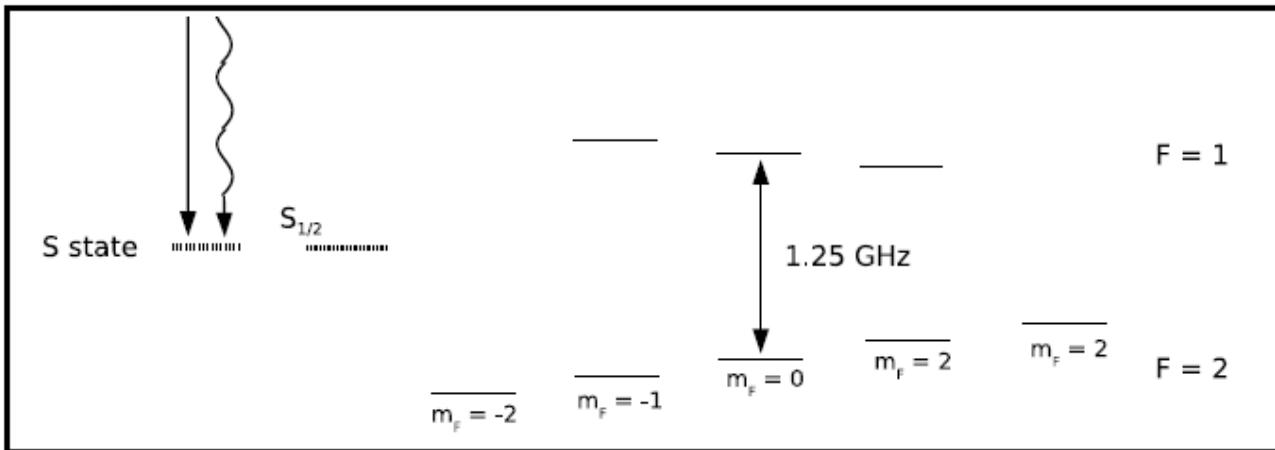
1. Long lifetime
2. Raman beams / microwaves
3. Odd isotopes of alkaline earth elements
4. Rabi frequency $\Omega_{RT} = \frac{\Omega_1 \Omega_2}{\Delta}$

Quantum information processing

Be-qubit
NIST

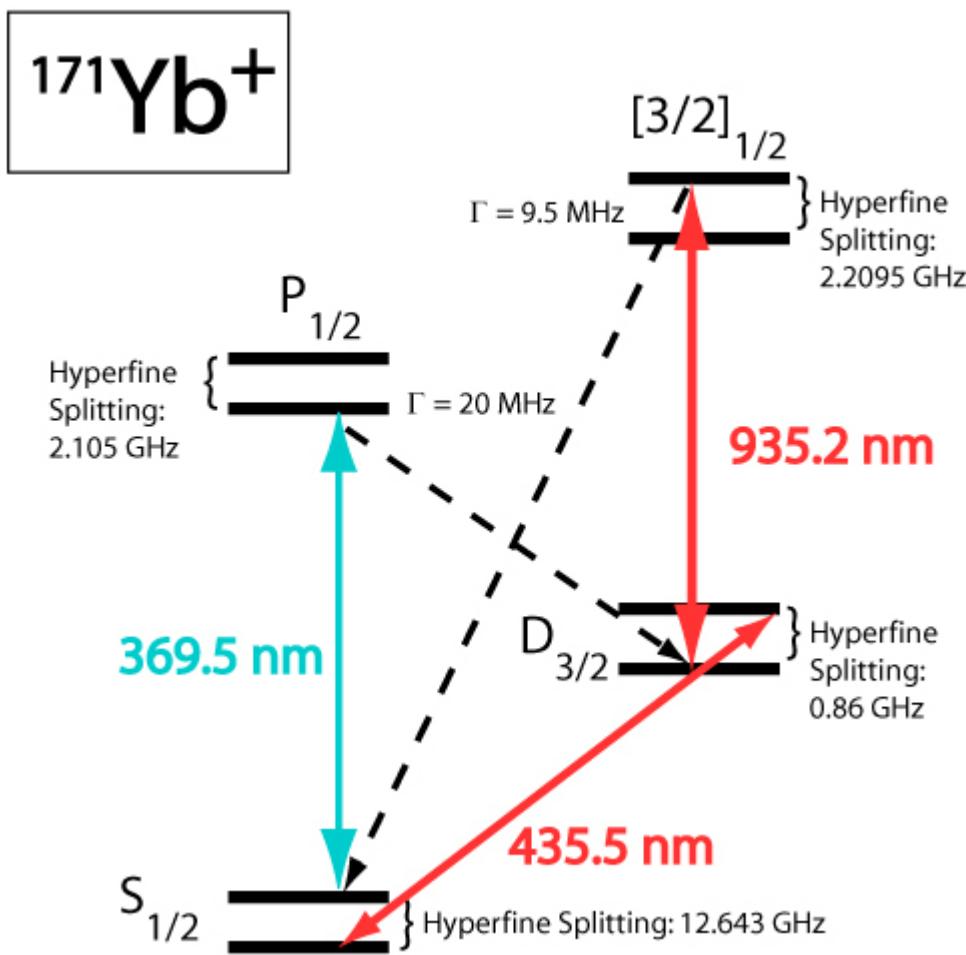


313 nm

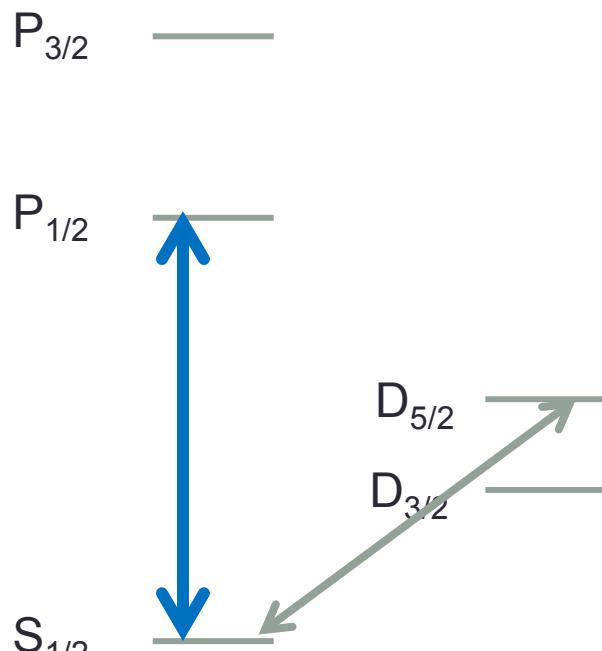


Quantum information processing

Yb-qubit
JQI



Quantum information processing

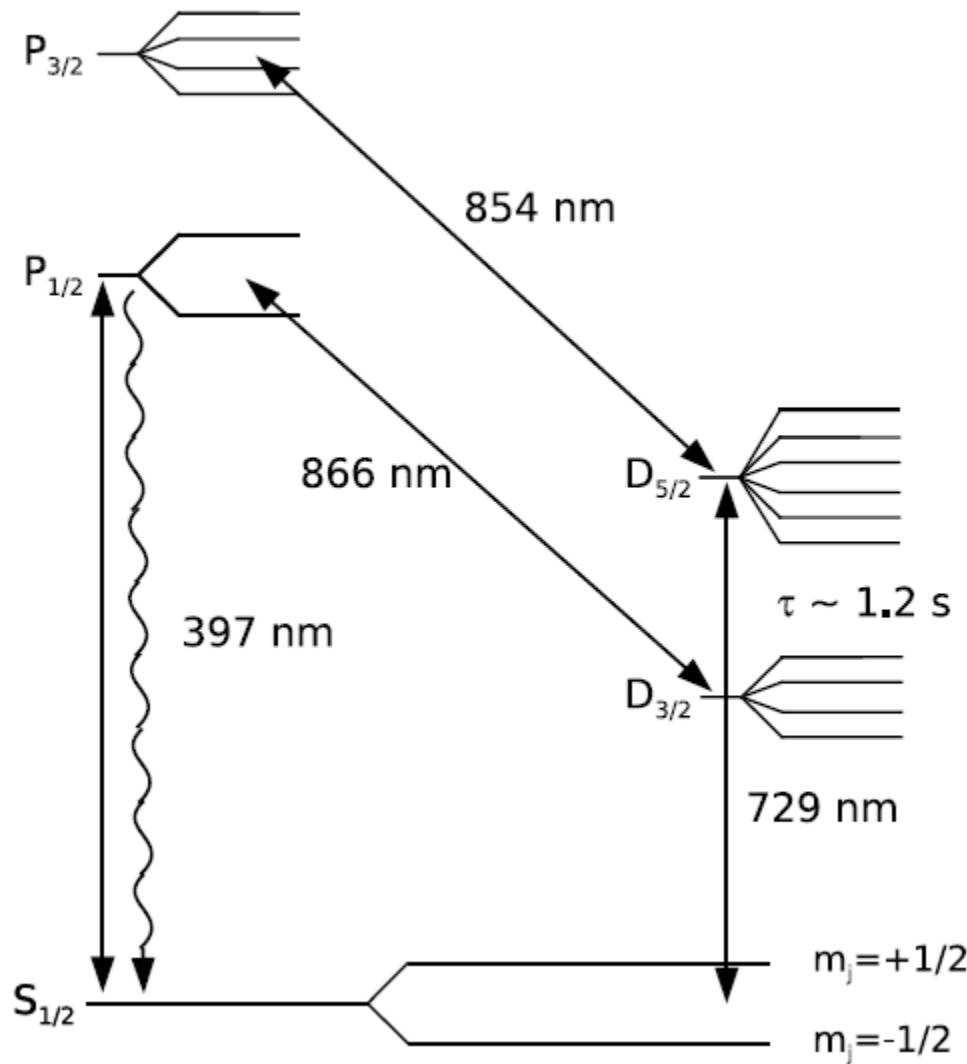


Optical Qubit:

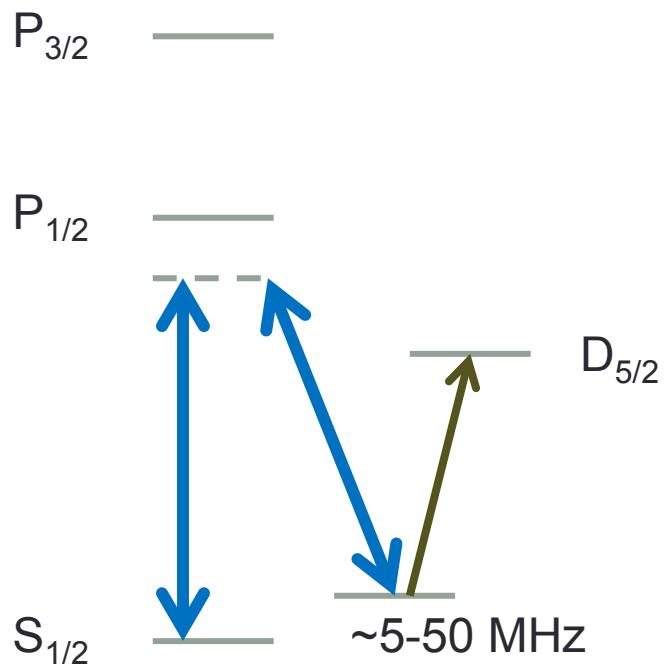
1. Long lifetime
2. Narrow linewidth lasers
3. even isotopes of alkaline earth elements
4. Rabi frequency $\Omega_{QT} = \frac{e k}{\hbar} \left\langle g \left| |\vec{r}| (\vec{E} \cdot \vec{r}) \right| e \right\rangle$

Quantum information processing

Ca qubit
Innsbruck



Quantum information processing

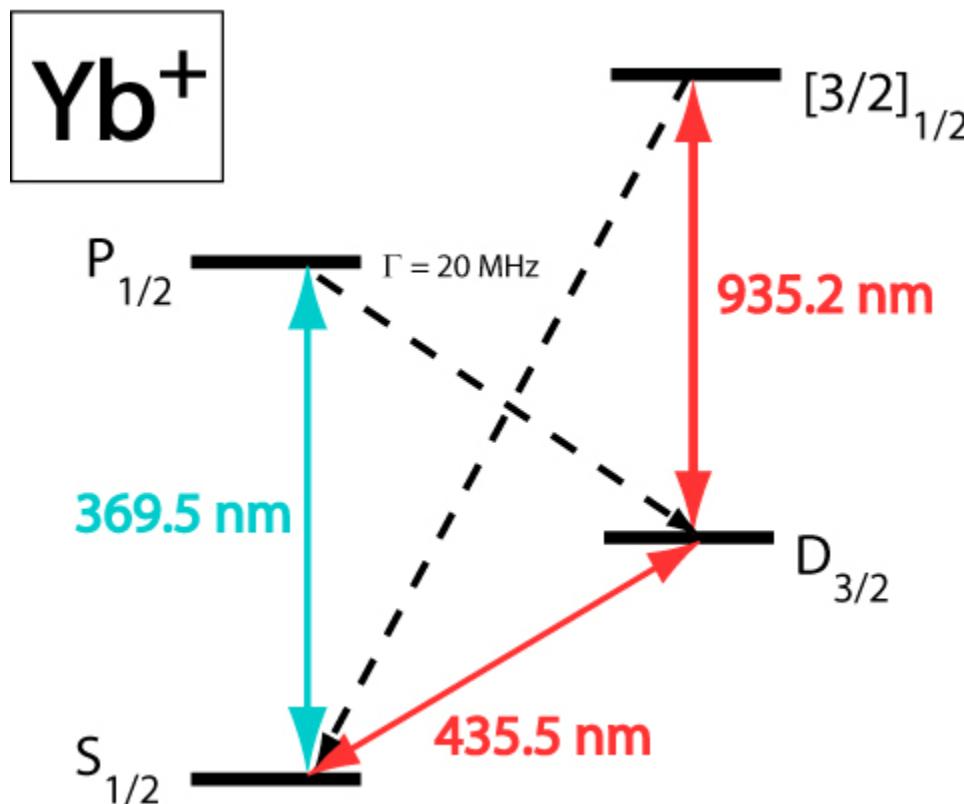


Zeeman Qubit:

1. Long lifetime
2. Raman lasers / Radio frequency excitation
3. Even/odd isotopes of alkaline earth elements
4. Susceptible to magnetic field noise

Quantum information processing

Yb qubit
Ulm University



Choice of qubit

1																			18
1 H 1.008	2																	2 He 4.0026	
3 Li 6.94	4 Be 9.0122																		
11 Na 22.990	12 Mg 24.305	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17			
19 K 39.098	20 Ca 40.078	21 Sc 44.956	22 Ti 47.867	23 V 50.942	24 Cr 51.996	25 Mn 54.938	26 Fe 55.845	27 Co 58.933	28 Ni 58.693	29 Cu 63.546	30 Zn 65.38	31 Ga 69.723	32 Ge 72.630	33 As 74.922	34 Se 78.97	35 Br 79.904	36 Kr 83.798		
37 Rb 85.468	38 Sr 87.62	39 Y 88.906	40 Zr 91.224	41 Nb 92.906	42 Mo 95.95	43 Tc (98)	44 Ru 101.07	45 Rh 102.91	46 Pd 106.42	47 Ag 107.87	48 Cd 112.41	49 In 114.82	50 Sn 118.71	51 Sb 121.76	52 Te 127.60	53 I 126.90	54 Xe 131.29		
55 Cs 132.91	56 Ba 137.33	57-71 *	72 Hf 178.49	73 Ta 180.95	74 W 183.84	75 Re 186.21	76 Os 190.23	77 Ir 192.22	78 Pt 195.08	79 Au 196.97	80 Hg 200.59	81 Tl 204.38	82 Pb 207.2	83 Bi 208.98	84 Po (209)	85 At (210)	86 Rn (222)		
87 Fr (223)	88 Ra (226)	89-103 #	104 Rf (265)	105 Db (268)	106 Sg (271)	107 Bh (270)	108 Hs (277)	109 Mt (276)	110 Ds (281)	111 Rg (280)	112 Cn (285)	113 Nh (286)	114 Fl (289)	115 Mc (289)	116 Lv (293)	117 Ts (294)	118 Og (294)		

* Lanthanide series

57 La 138.91	58 Ce 140.12	59 Pr 140.91	60 Nd 144.24	61 Pm (145)	62 Sm 150.36	63 Eu 151.96	64 Gd 157.25	65 Tb 158.93	66 Dy 162.50	67 Ho 164.93	68 Er 167.26	69 Tm 168.93	70 Yb 173.05	71 Lu 174.97
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Actinide series

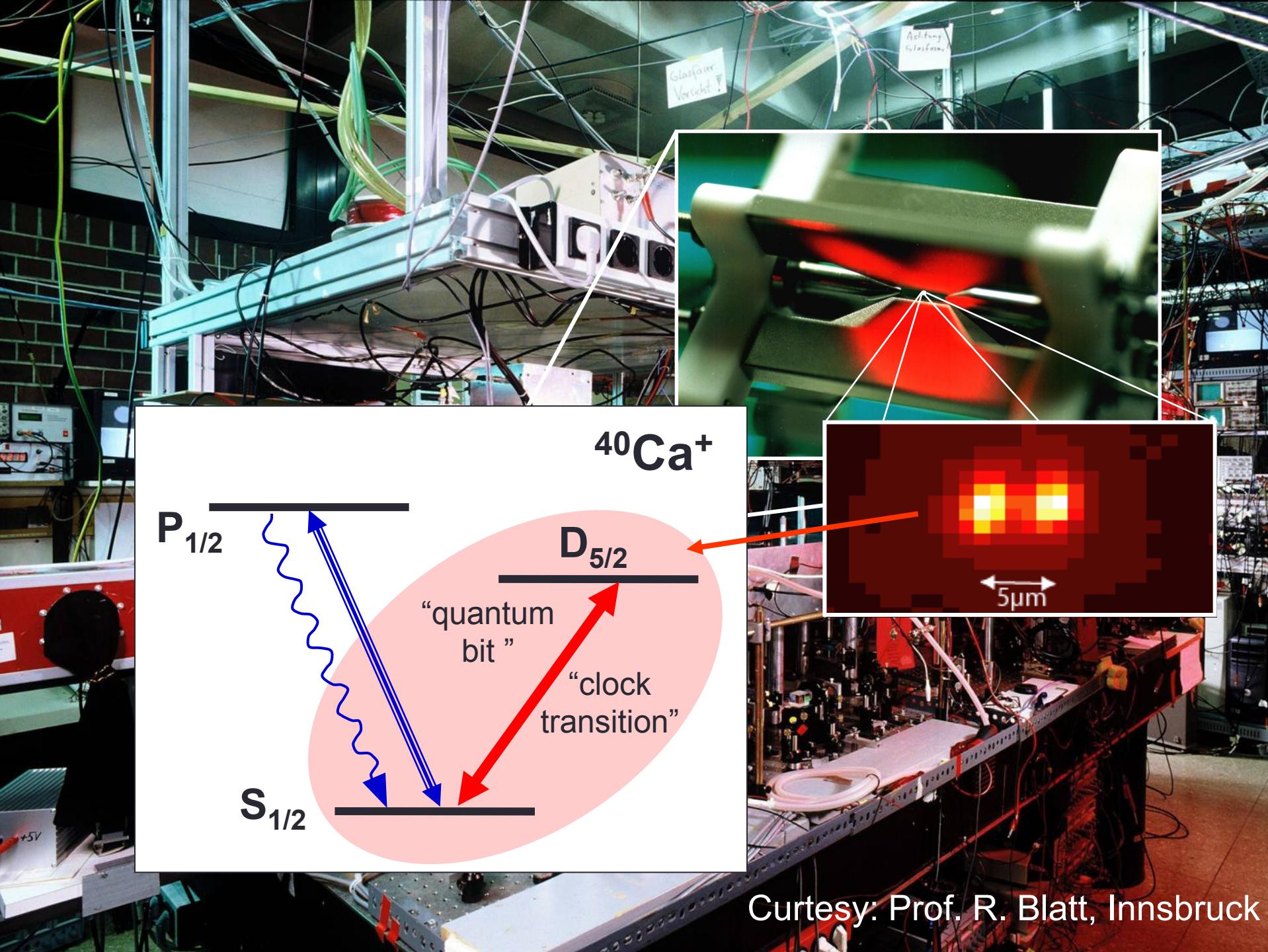
89 Ac (227)	90 Th 232.04	91 Pa 231.04	92 U 238.03	93 Np (237)	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (262)
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Choice of qubit

1. The relevant transitions should have accessible laser wavelength
2. Light atoms have higher Lamb-Dicke parameter
3. Suitable qubit transition

$$\eta = k \sqrt{\frac{\hbar}{2m\nu_{sec}}} = \frac{2\pi}{\lambda} \sqrt{\frac{\hbar}{2m\nu_{sec}}}$$



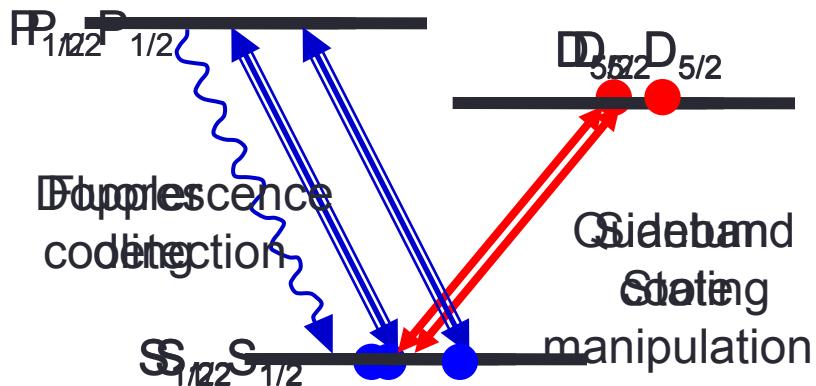


Courtesy: Prof. R. Blatt, Innsbruck

Quantum information processing

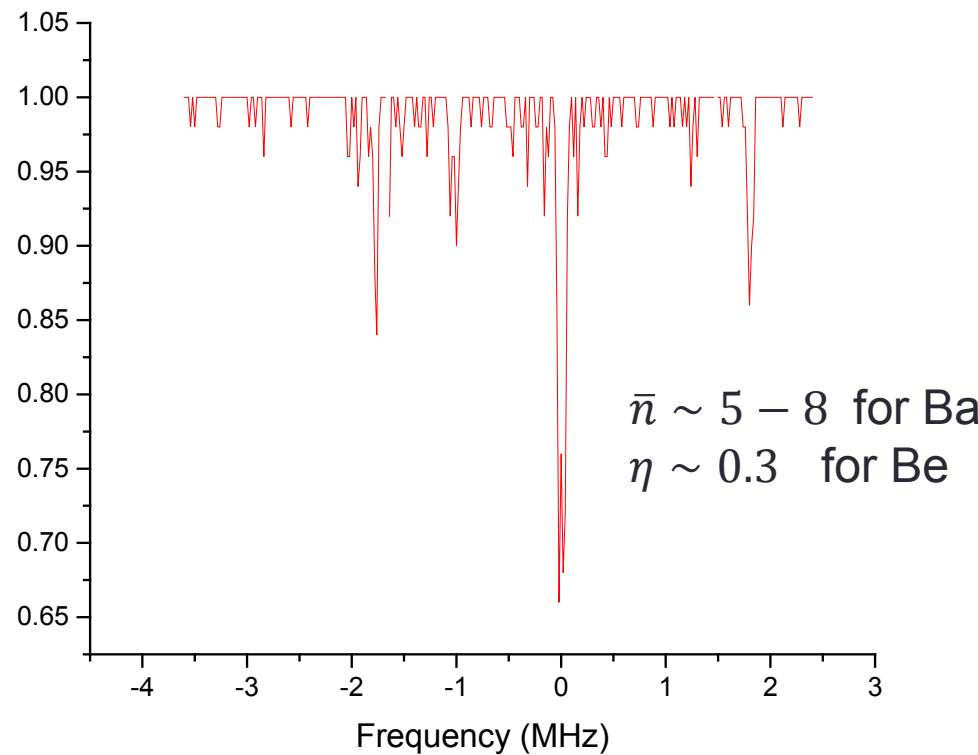
The recipe

- Initialization
- Quantum state manipulation
- Quantum state measurement



Quantum information processing

Initialization

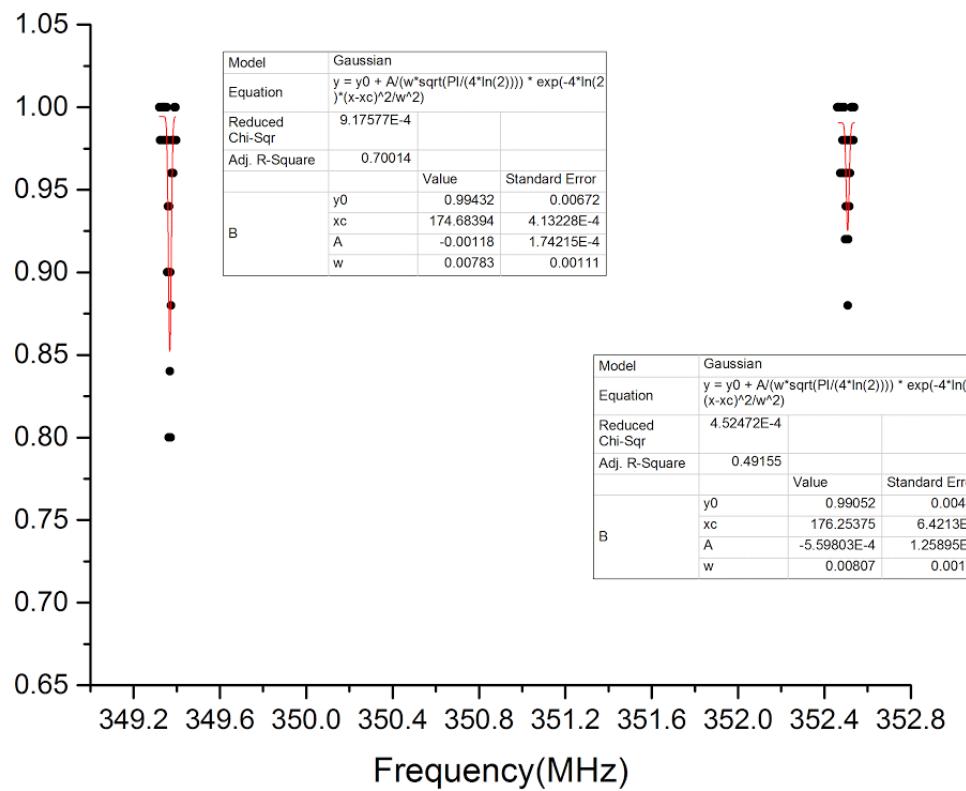


Step1: Doppler cooling 0.5-1 ms
Goal: reach $\eta \ll 1$

$T = 1$ ms

Quantum information processing

Initialization



Step2: Side-band cooling 1-5 ms

Goal: reach $\bar{n} \sim 0.1$

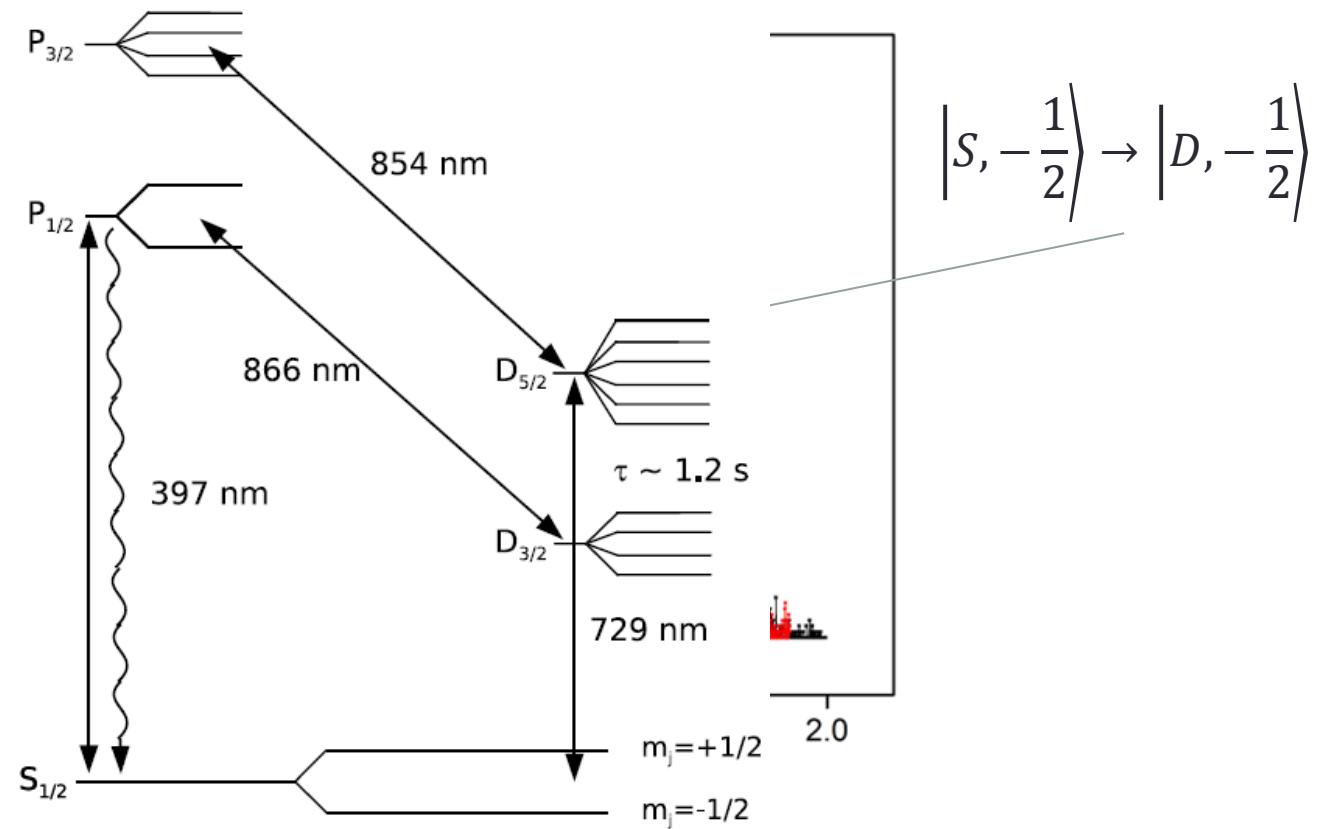
$T = 1+5$ ms



Quantum information processing

Initialization

$$\left|S, +\frac{1}{2}\right\rangle \rightarrow \left|D, +\frac{1}{2}\right\rangle$$

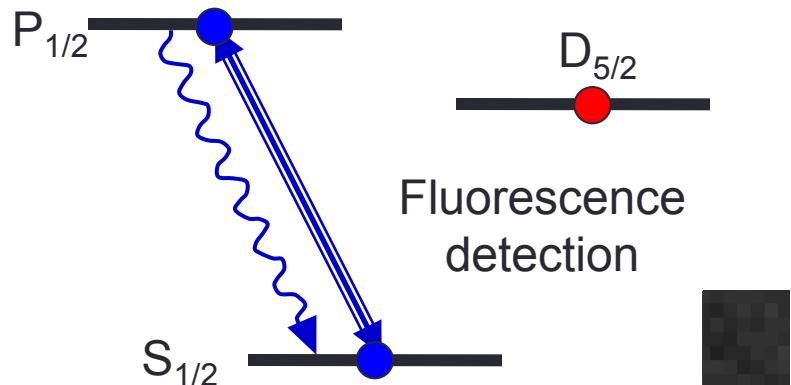


Step2: Optical pumping $10 - 100 \mu\text{s}$
 Goal: population in $|g, 0\rangle \geq 99.99\%$

$T = 1+5+0.1 \text{ ms}$

Quantum information processing

Measurement

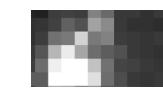


Two ions:

5μm



|SS⟩



|SD⟩



|DS⟩



|DD⟩

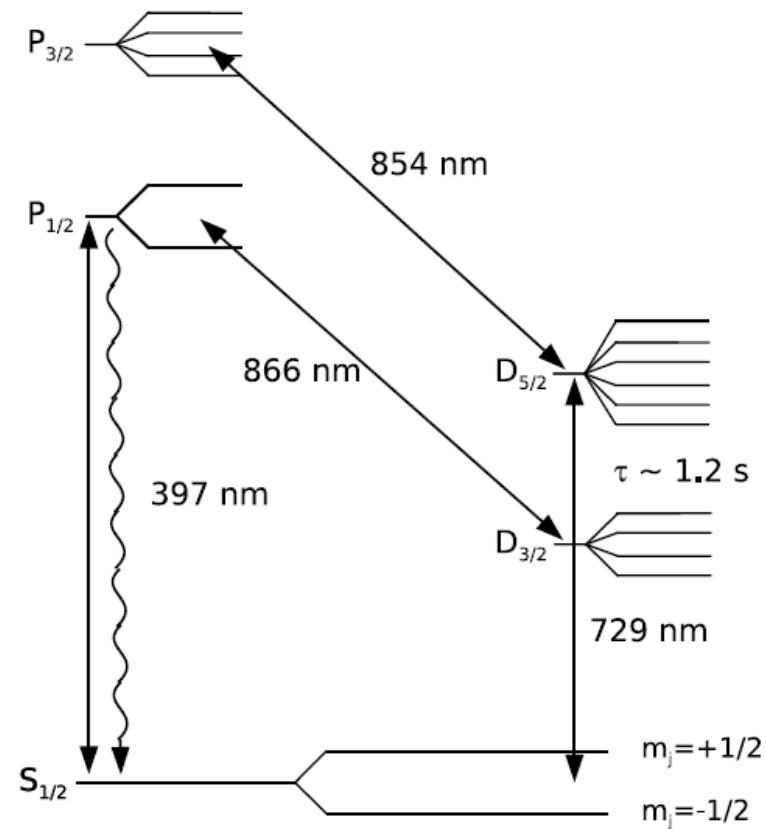
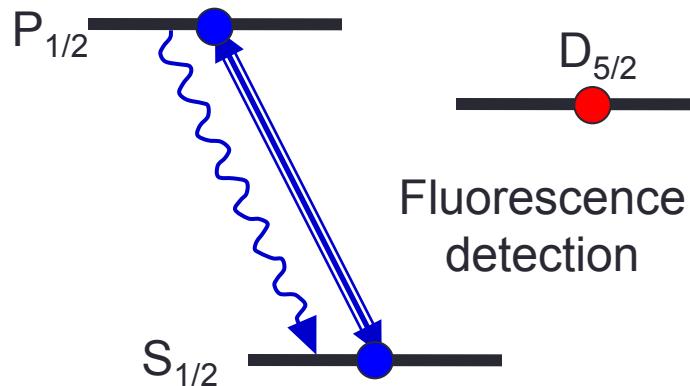
50 experiments / s

Repeat experiments
100-200 times



Quantum information processing

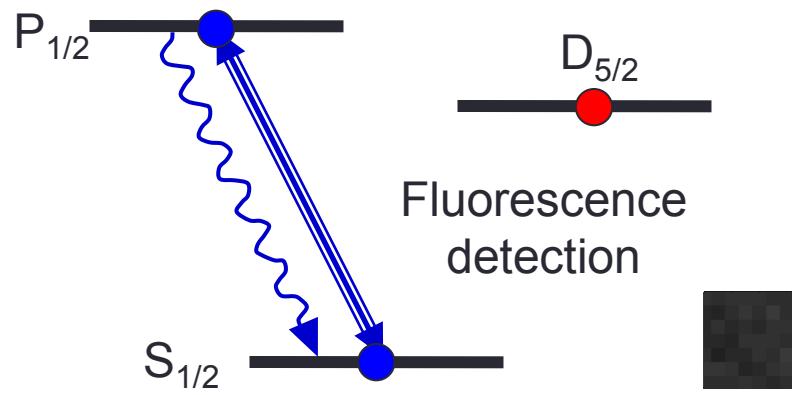
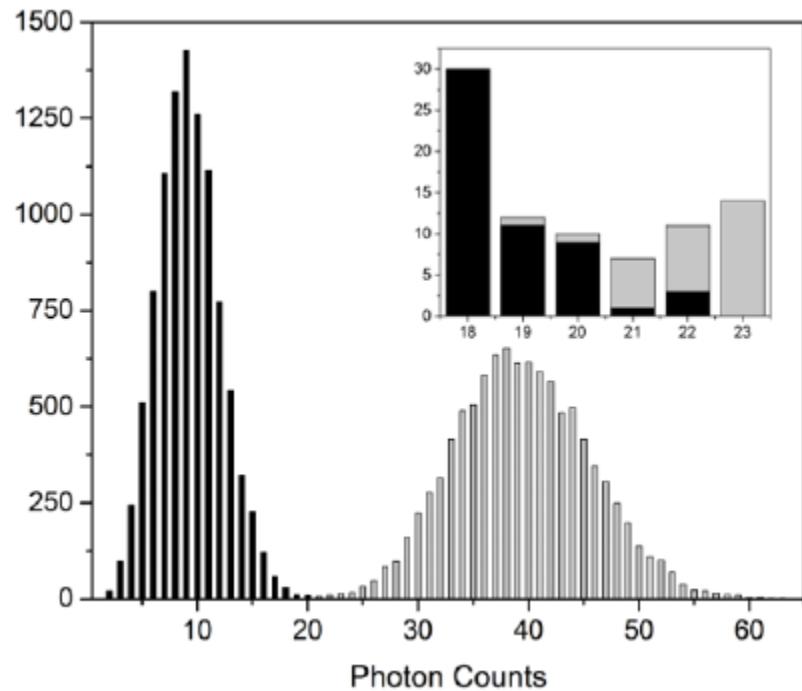
Measurement / state detection



Problem 10: Considering the optical dipole transition in Ca ion, calculate the number of photons arriving the CCD camera with an overall collection efficiency of 0.1%

Quantum information processing

Measurement / state discrimination



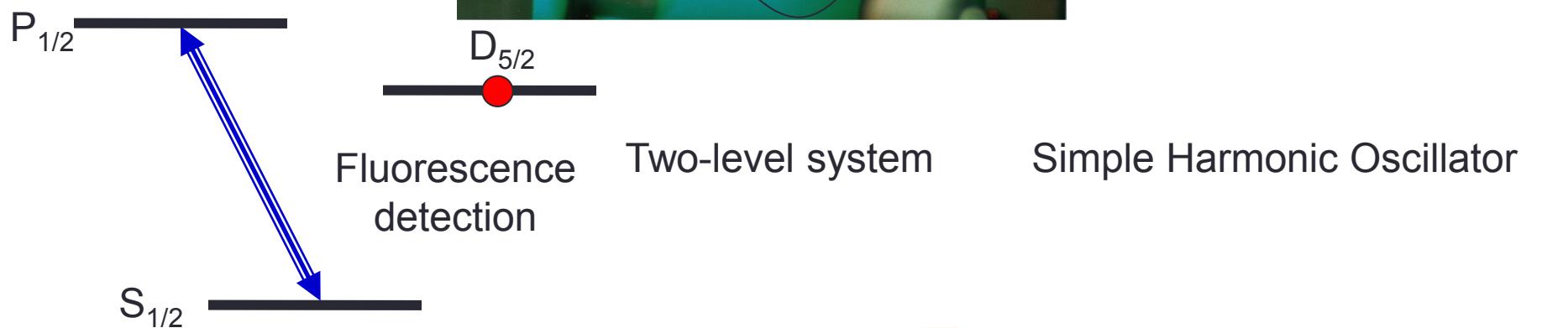
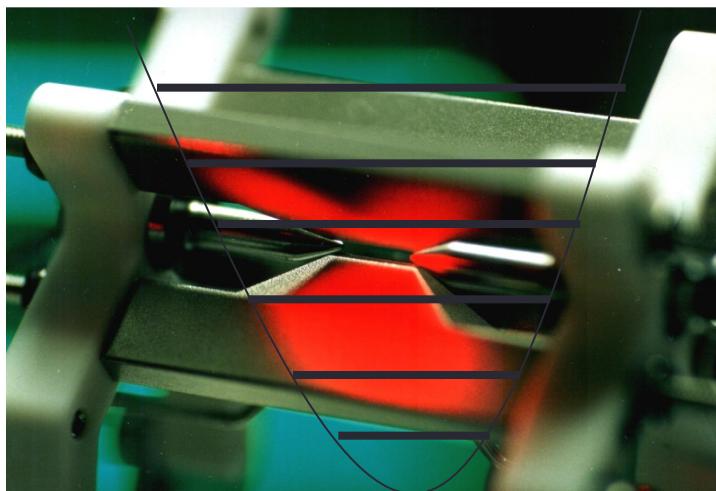
Typical error in state discrimination < 0.1% within a time of 1 ms

$$T = 1+5+0.1+1 \text{ ms}$$

Quantum information processing

Q-state manipulation

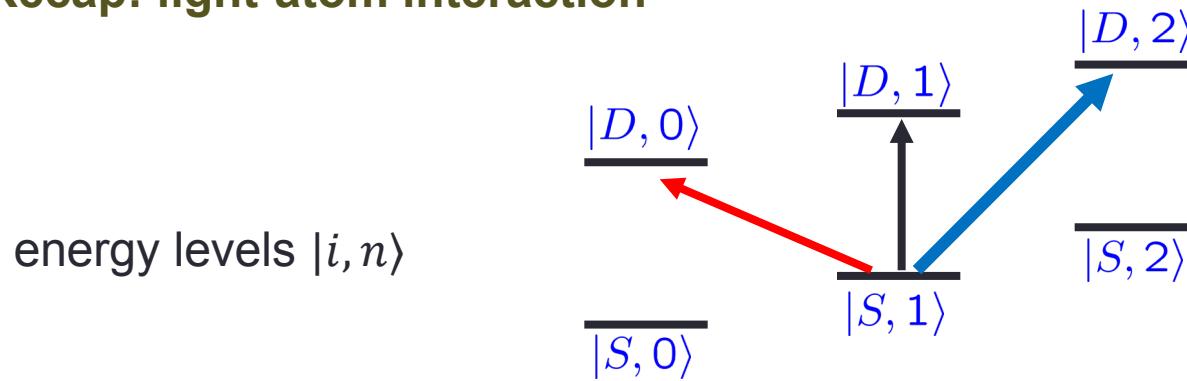
Summarize the model:



Quantum information processing

Q-state manipulation

Recap: light-atom interaction



$$1. \delta = 0, H_{car} = (h / 4 * \pi) \Omega_0 (\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi})$$

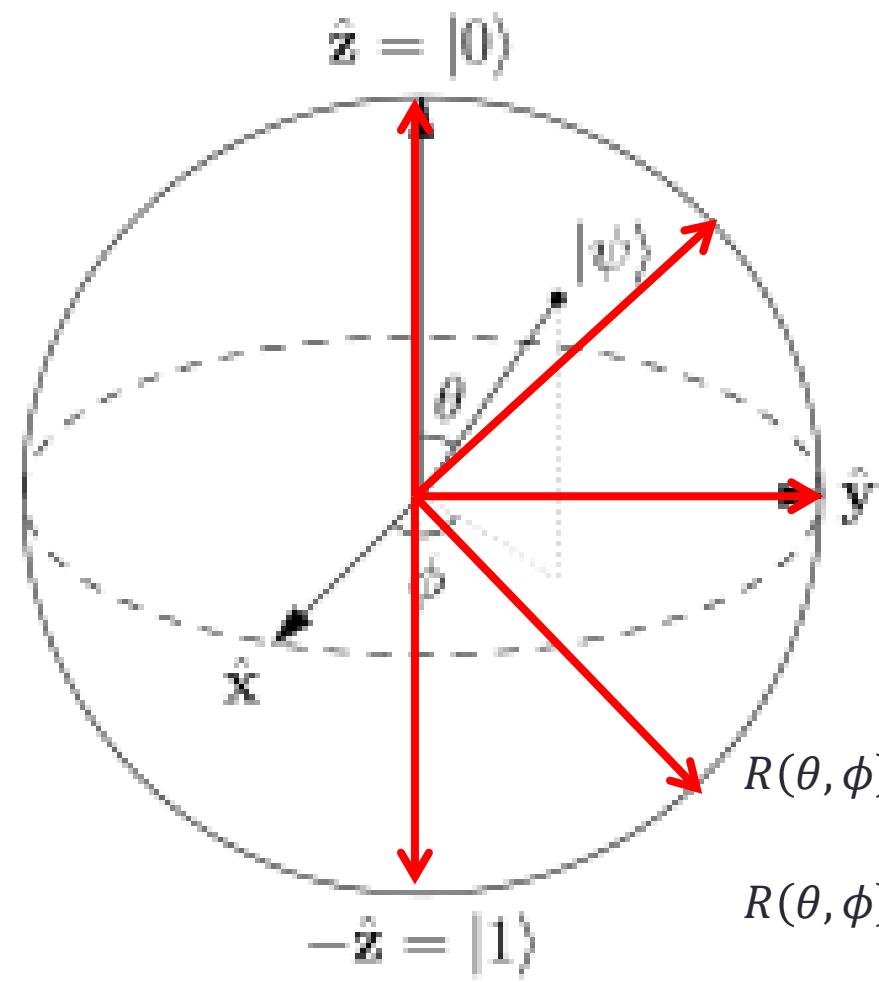
$$2. \delta = -\nu, H_{rsb} = (h / 4 * \pi) \Omega_0 \eta (a \sigma_+ e^{i\phi} + a^\dagger \sigma_- e^{-i\phi})$$

$$3. \delta = \nu, H_{bsb} = (h / 4 * \pi) \Omega_0 \eta (a^\dagger \sigma_+ e^{i\phi} + a \sigma_- e^{-i\phi})$$



Quantum information processing

Q-state manipulation – single qubit



$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

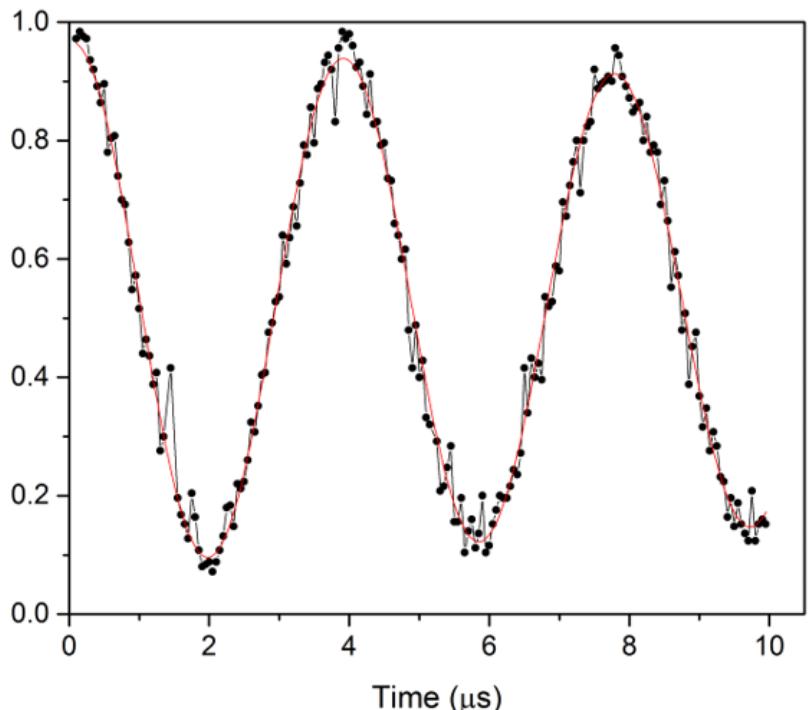
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$R(\theta, \phi) = \exp\left(\frac{i\theta}{2}(e^{i\phi} \sigma_+ + e^{-i\phi} \sigma_-)\right)$$

$$R(\theta, \phi) = I \cos\left(\frac{\theta}{2}\right) + i(\sigma_x \cos(\phi) - \sigma_y \sin(\phi)) \sin\left(\frac{\theta}{2}\right)$$

Quantum information processing

Q-state manipulation – single qubit



$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

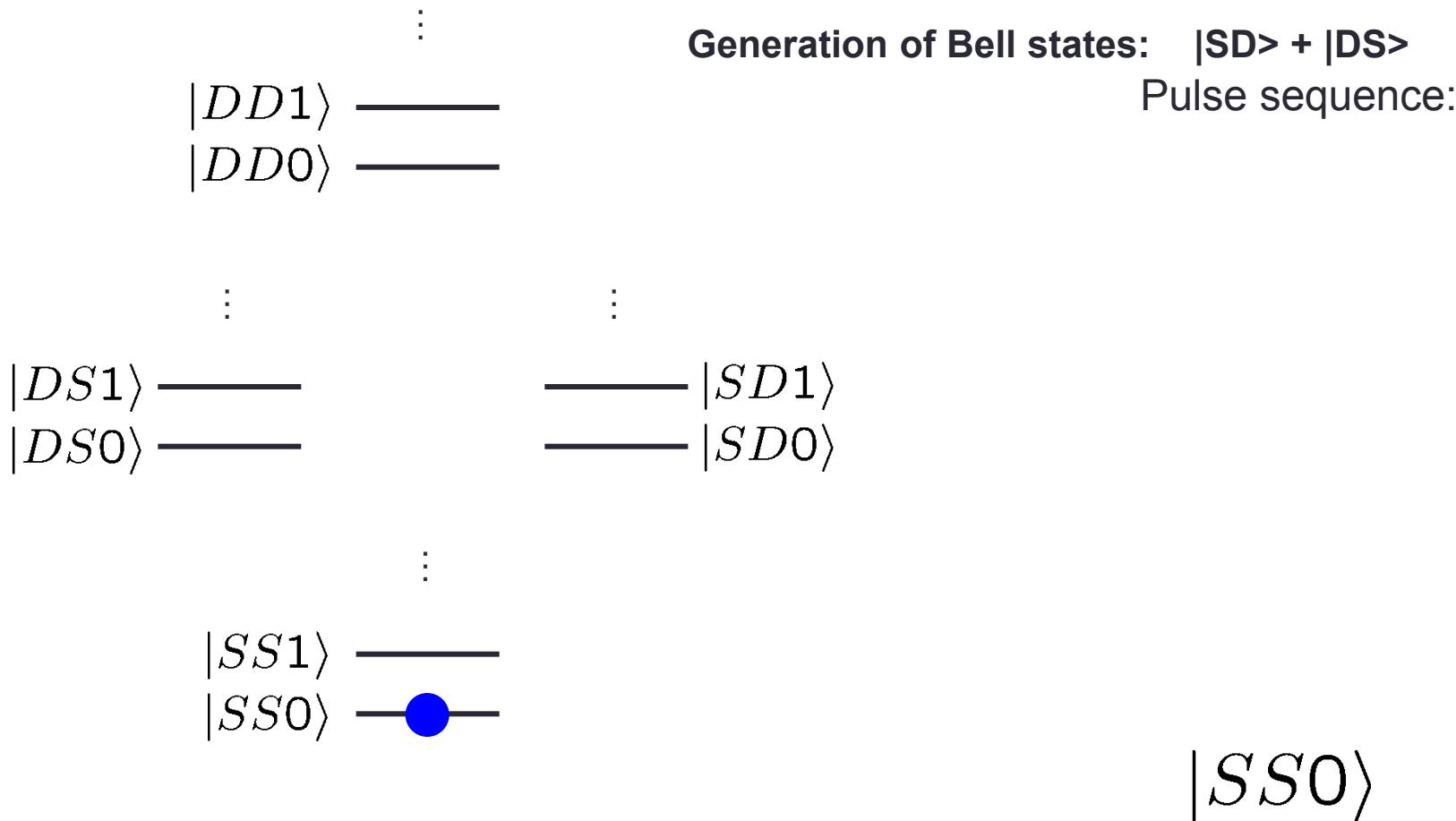
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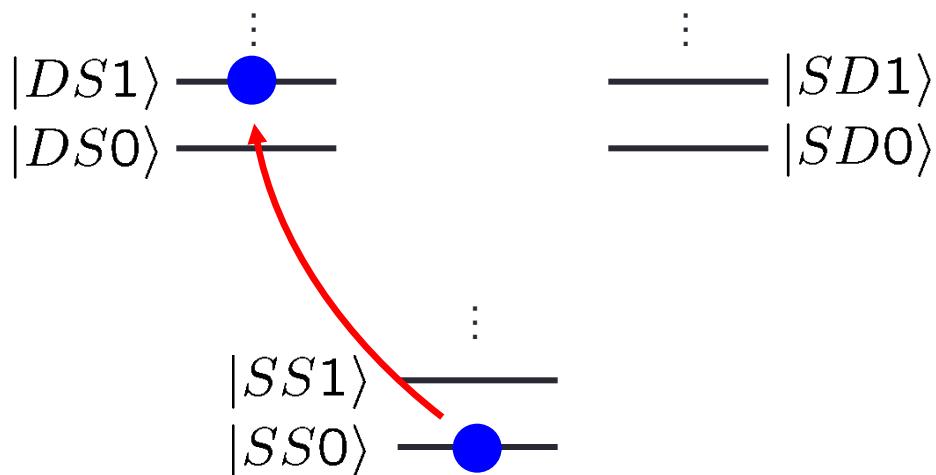
Generation of Bell state



Generation of Bell state

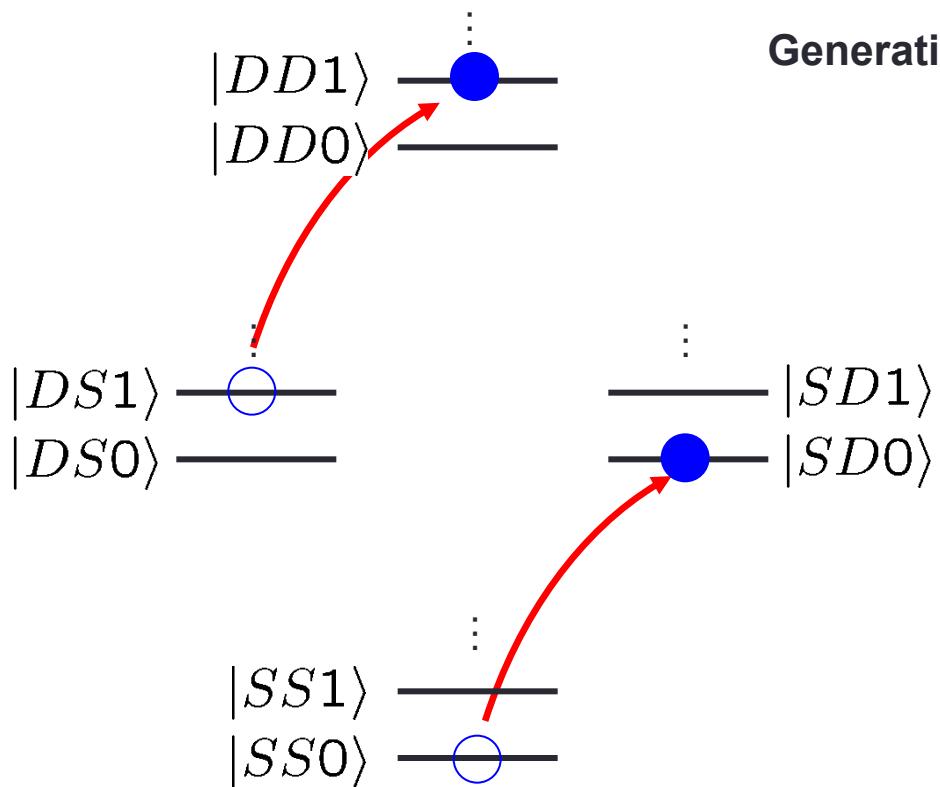
$|DD1\rangle$ —
 $|DD0\rangle$ —

Generation of Bell states: $|SD\rangle + |DS\rangle$
Pulse sequence:
Ion 1: $\pi/2$, blue sideband



$$|SS0\rangle + |DS1\rangle$$

Generation of Bell state



Generation of Bell states: $|SD> + |DS>$

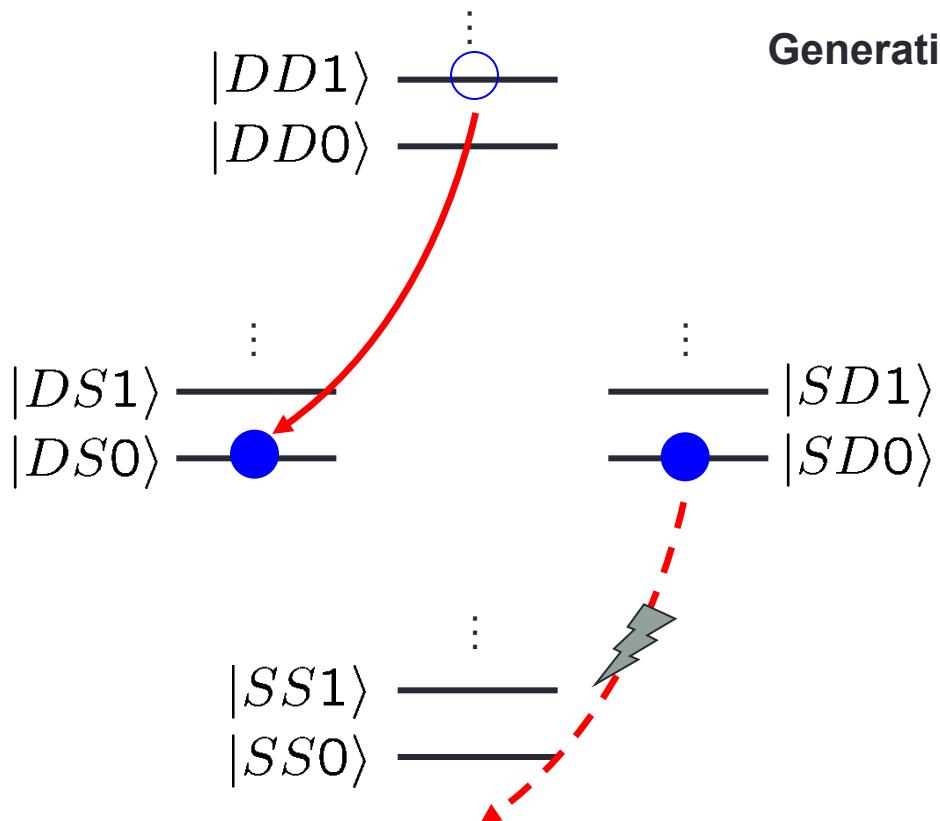
Pulse sequence:

Ion 1: $\pi/2$, blue sideband

Ion 2: π , carrier

$$|SD0\rangle + |DD1\rangle$$

Generation of Bell state



Generation of Bell states: $|SD\rangle + |DS\rangle$

Pulse sequence:

Ion 1: $\pi/2$, blue sideband

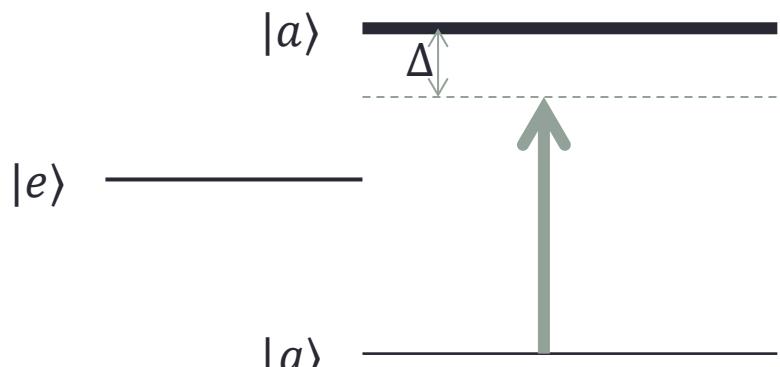
Ion 2: π , carrier

Ion 2: π , blue sideband

$$(|SD\rangle + |DS\rangle)|0\rangle$$

Quantum phase

AC Stark shift



$$\frac{\Delta E}{\hbar} = \frac{\Omega^2}{2\Delta}$$

$$a|e\rangle + b|g\rangle \rightarrow |e\rangle + re^{-\frac{\Delta E}{\hbar}t}|g\rangle$$

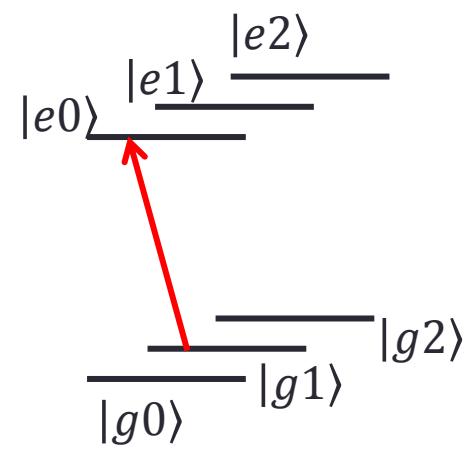
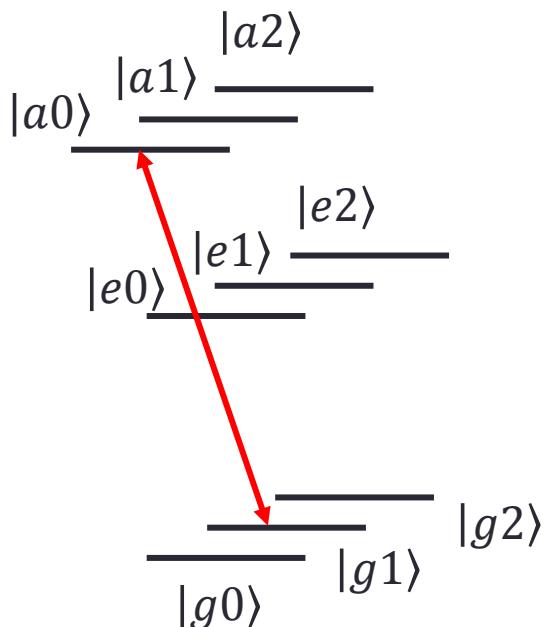
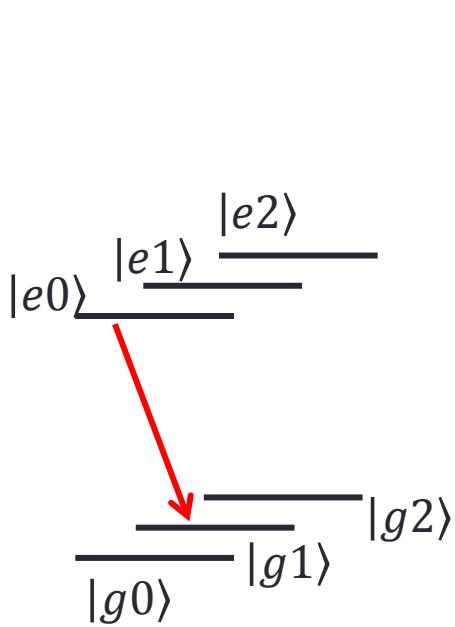
Therefore we can use AC stark shift to generate relative phase



Universal gate – C-NOT

Q-state manipulation – 2 qubit (original CZ gate)

2π on the red side band



$$|g1\rangle \rightarrow -|g1\rangle$$

Ion #1

Ion #2

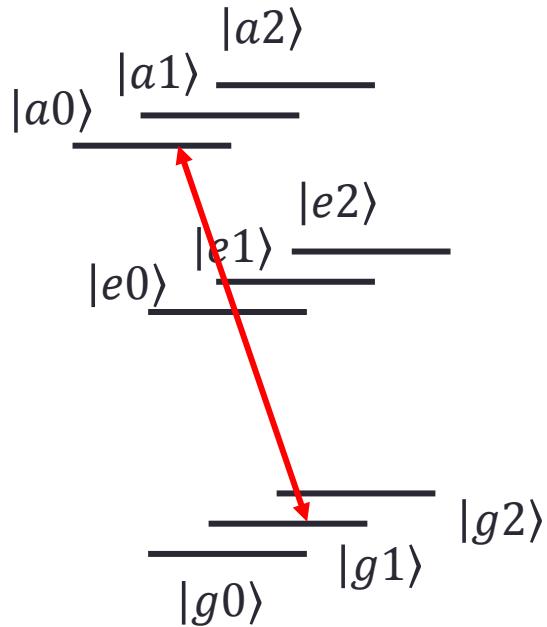
Ion #1



Universal gate – C-NOT

Q-state manipulation – 2 qubit (original CZ gate)

2π on the red side band



Phase gate

$$|g1\rangle \rightarrow -|g1\rangle$$

$$|g0\rangle \rightarrow |g0\rangle$$

$$|e1\rangle \rightarrow |e1\rangle$$

$$|e0\rangle \rightarrow |e0\rangle$$

Ion #2

Universal gate – C-NOT

Q-state manipulation – 2 qubit (original CZ gate)

Phase gate

$$|ee0\rangle \rightarrow -|ee0\rangle$$

$$|eg0\rangle \rightarrow |eg0\rangle$$

$$|ge0\rangle \rightarrow |ge0\rangle$$

$$|gg0\rangle \rightarrow |gg0\rangle$$

$$R\left(\frac{\pi}{2}, \phi_i\right)$$



$$|ee0\rangle \rightarrow |eg0\rangle$$

$$|eg0\rangle \rightarrow |ee0\rangle$$

$$|ge0\rangle \rightarrow |ge0\rangle$$

$$|gg0\rangle \rightarrow |gg0\rangle$$



a. $|g\rangle_m|g\rangle_n|0\rangle-$

$\rightarrow|g\rangle_m|g\rangle_n|0\rangle$

b. $|g\rangle_m|e_0\rangle_n|0\rangle-$

C-NOT gate

$\rightarrow|g\rangle_m|e_0\rangle_n|0\rangle$

c. $|e_0\rangle_m|g\rangle_n|0\rangle-$

$V^{1/2}_n(\Pi/2)P\text{Gate}V^{1/2}_n(-\Pi/2)$

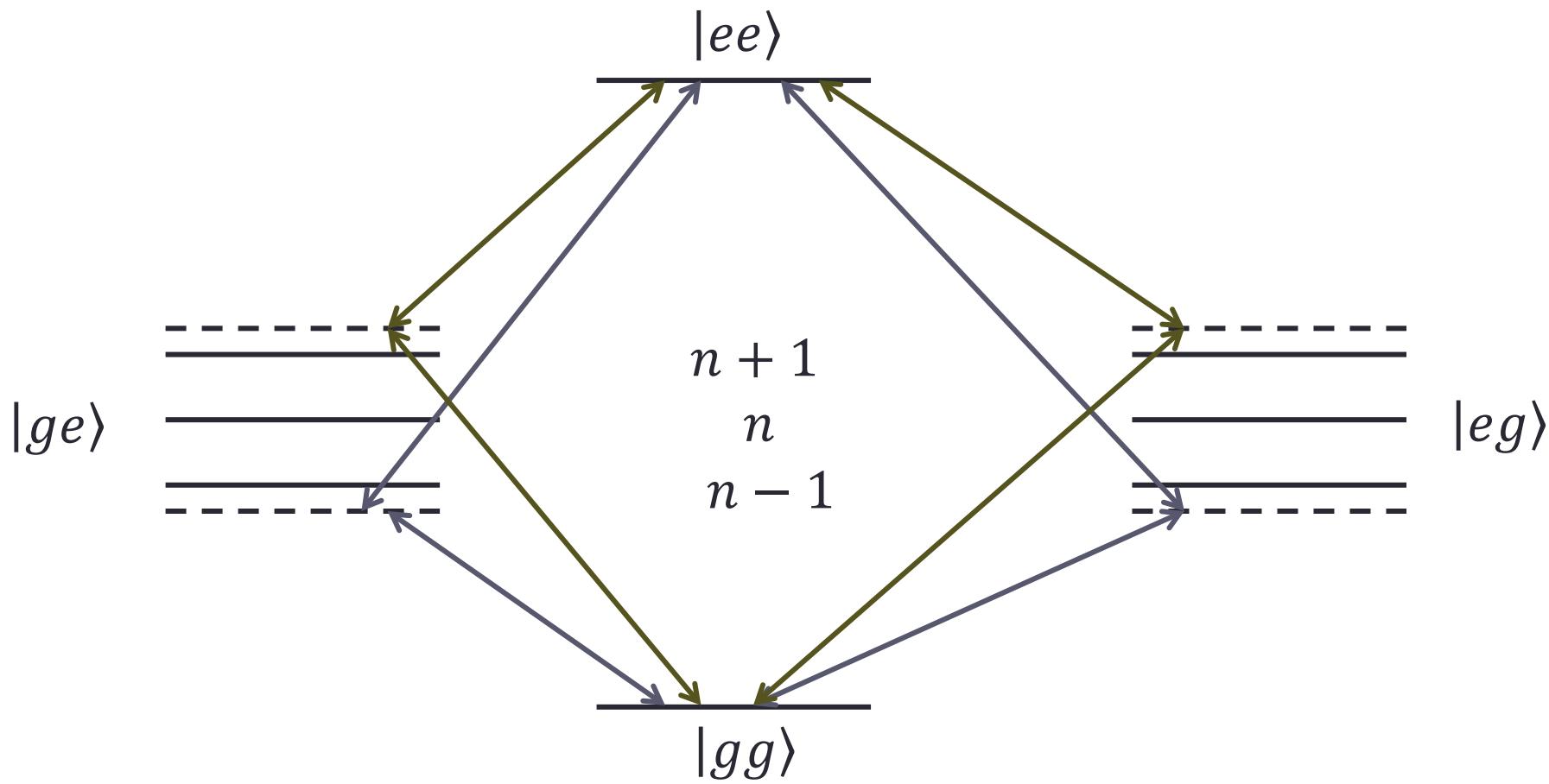
$\rightarrow|e_0\rangle_m|e_0\rangle_n|0\rangle$

d. $|e_0\rangle_m|e_0\rangle_n|0\rangle-$

$\rightarrow|e_0\rangle_m|g\rangle_n|0\rangle$

Universal gate

Q-state manipulation – 2 qubit (A. Sørensen & K. Mølmer)



Universal gate

Q-state manipulation – 2 qubit (A. Sørensen & K. Mølmer)

$$|ee\rangle \rightarrow (|ee\rangle + i|gg\rangle)/\sqrt{2}$$

$$|gg\rangle \rightarrow (|gg\rangle + i|ee\rangle)/\sqrt{2}$$

$$|eg\rangle \rightarrow (|eg\rangle + i|ge\rangle)/\sqrt{2}$$

$$|ge\rangle \rightarrow (|ge\rangle + i|eg\rangle)/\sqrt{2}$$

Consider new basis as $|\pm\rangle_i = (|e\rangle_i \pm |g\rangle_i)/\sqrt{2}$

Universal gate

Q-state manipulation – 2 qubit (A. Sørensen & K. Mølmer)

$$|++\rangle \rightarrow |++\rangle$$

$$|--\rangle \rightarrow |--\rangle$$

$e^{\frac{i\pi}{4}}$ global phase ignored

$$|+-\rangle \rightarrow i|+-\rangle$$

$$|-+\rangle \rightarrow i|-+\rangle$$

It is universal along with single qubit rotation



Universal gate

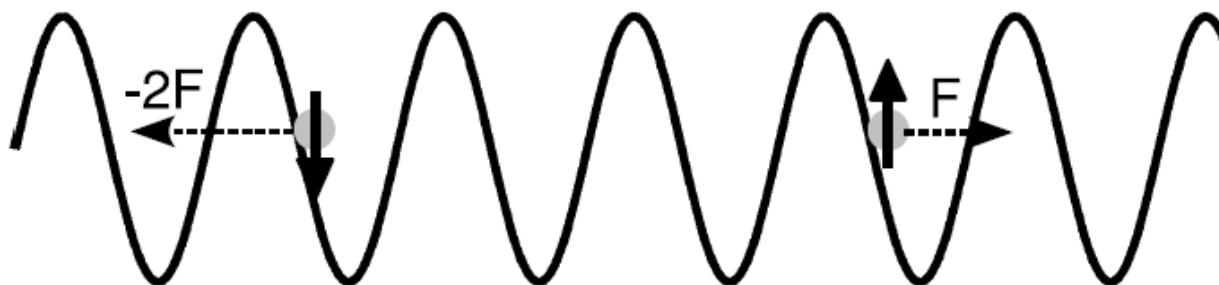
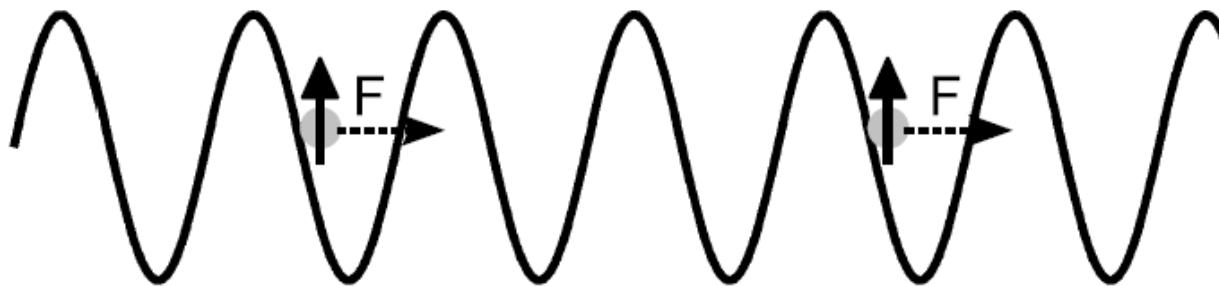
Geometric phase gates – single qubit

For single qubit these gates are usually produced by adiabatic rotation in Bloch sphere giving rise to Berry's phase.

For Universal operation, one needs both Abelian and non-Abelian phases

Universal gate

Geometric phase gates – two qubit



Universal gate

Challenges in QC

Suppose we want to drive the sideband at a rate $f\omega_s$ (similar to trap frequency) the coupling strength required is

$$\Omega = \frac{\Omega_+}{\eta} = \frac{f\omega_s}{\eta}$$

At this rate the AC Stark shift of the qubit will be

$$\frac{\Delta E}{\hbar} = \frac{\Omega^2}{2\Delta} = \frac{1}{2\omega_s} \frac{f\omega_s\Omega_+}{\eta^2} = \frac{f}{2\eta^2} \Omega_+$$

This means the phase evolution due to AC Stark shift becomes comparable to Ω_+ already for $f = \eta^2$

Therefore (1) AC Stark shift (2) off resonant coupling $\sim \frac{f^2}{\eta^2}$ are problematic

Universal gate

Challenges in QC

1. Gate fidelity
2. Gate time
3. Scalability

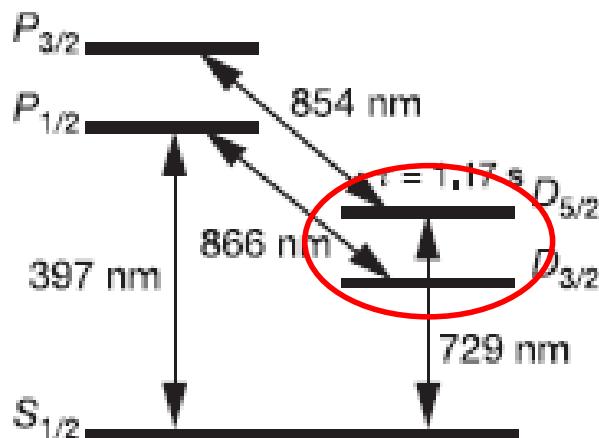
References for day 4:

(look also for references within the review)

1. D. Leibfried et al. Rev. Mod. Phys. 75, 281 (2003)
2. Molmer and Sorensen Phys. Rev. Lett. 82, 1835 (1999)
3. D. Kielpinski et al. Nature 417, 709 (2002)

Quantum metrology

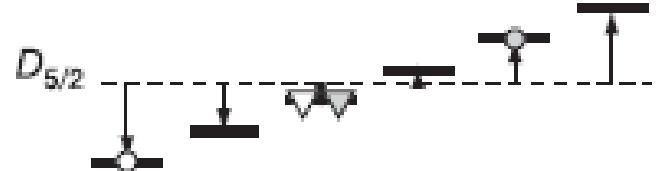
Entanglement in metrology: an example



Ca atomic levels

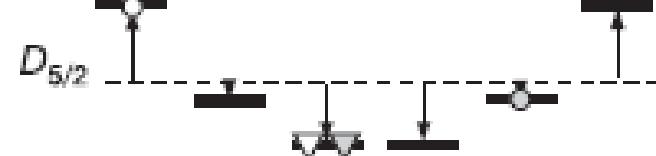
Application of magnetic field

$$m = -5/2 \quad -3/2 \quad -1/2 \quad +1/2 \quad +3/2 \quad +5/2$$



Calcium atomic level when trapped

$$m = -5/2 \quad -3/2 \quad -1/2 \quad +1/2 \quad +3/2 \quad +5/2$$

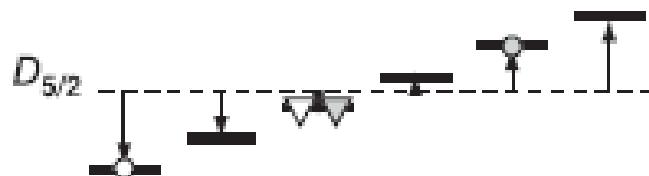


C. F. Roos et al. Nature 443 316 (2006)

Quantum metrology

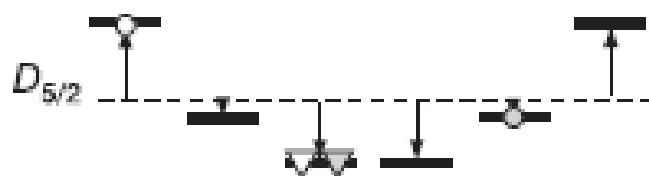
Entanglement in metrology: an example

b $m = -5/2 \quad -3/2 \quad -1/2 \quad +1/2 \quad +3/2 \quad +5/2$



$$\hbar\Delta\nu = \frac{1}{4} \frac{dE_z}{dz} \Theta(D, j) \frac{j(j+1) - 3m_j^2}{j(2j-1)} (3\cos^2\beta - 1)$$

c $m = -5/2 \quad -3/2 \quad -1/2 \quad +1/2 \quad +3/2 \quad +5/2$



Quadrupole shift measurement in Ca^+

C. F. Roos et al. Nature 443 316 (2006)

Quantum metrology

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|u_1\rangle|u_2\rangle + |v_1\rangle|v_2\rangle)$$

initial state

$$|\psi(\tau)\rangle = \frac{1}{\sqrt{2}}(|u_1\rangle|u_2\rangle + e^{i\lambda_\phi\tau}|v_1\rangle|v_2\rangle)$$

evolution of the state with time

$$\lambda_\phi = \frac{[(E_{u1}+E_{u2})-(E_{v1}+E_{v2})]}{\hbar}$$

measuring the phase provides
information about the energy difference

De-coherence free sub-space (DFS) chosen as

$$|\psi(\tau)\rangle = \frac{1}{\sqrt{2}}\left(\left|-\frac{5}{2}\right\rangle\left|+\frac{3}{2}\right\rangle + \left|-\frac{1}{2}\right\rangle\left|-\frac{1}{2}\right\rangle\right)$$

Not affected by magnetic field fluctuations



Quantum metrology

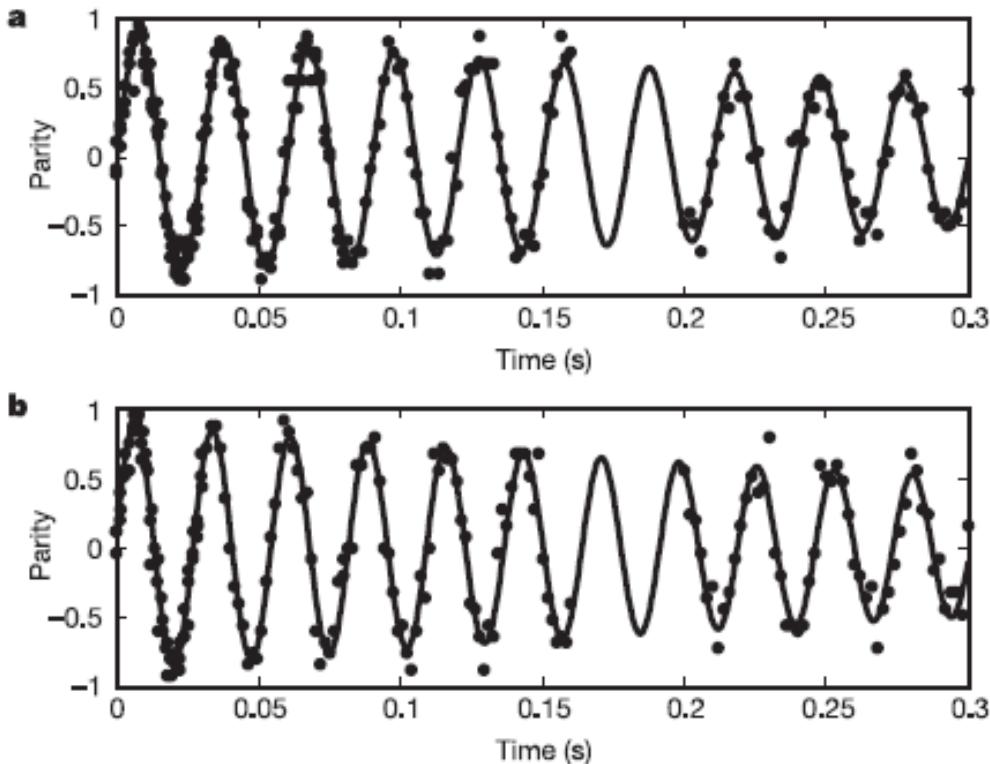
Entanglement in metrology: an example

Parity measurement:

$$P = \hat{P}_{++} + \hat{P}_{--} - \hat{P}_{+-} - \hat{P}_{-+}$$

$\hat{P}_{\pm\pm}$ implies projection on to
 $|\pm\rangle_1 \times |\pm\rangle_2$ where

$$|\pm\rangle_{k=1,2} = \frac{1}{\sqrt{2}}(|u_k\rangle \pm |v_k\rangle)$$



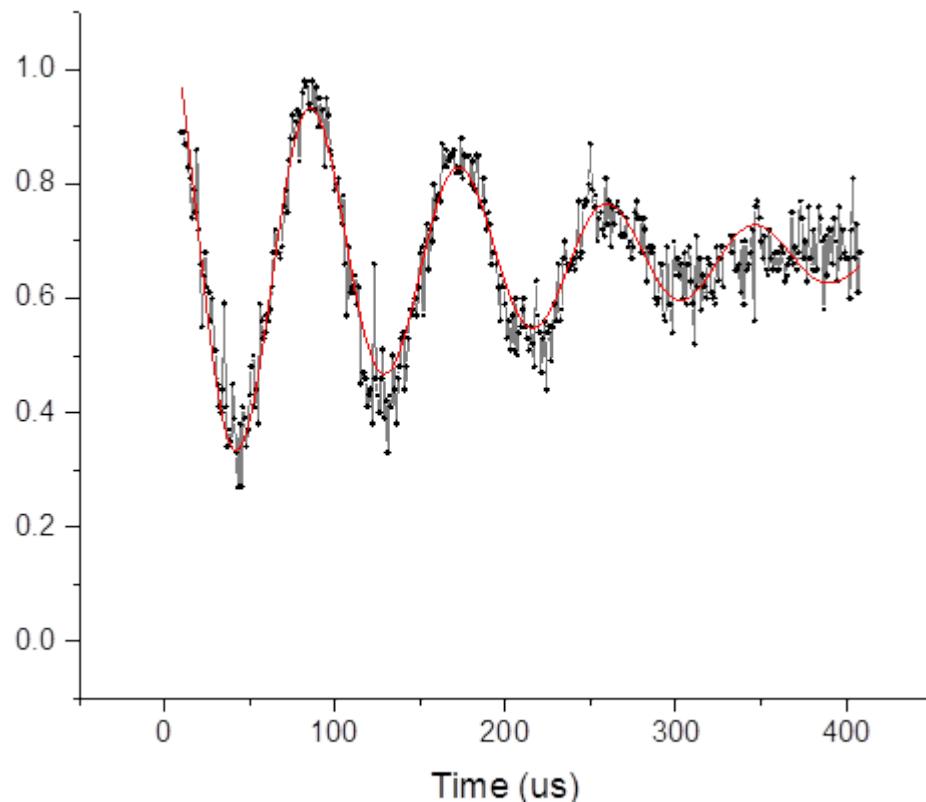
Quadrupole shift measurement in Ca^+
C. F. Roos et al. Nature 443 316 (2006)

Quantum metrology

Illness of magnetic field fluctuation: an example

Parity measurement:

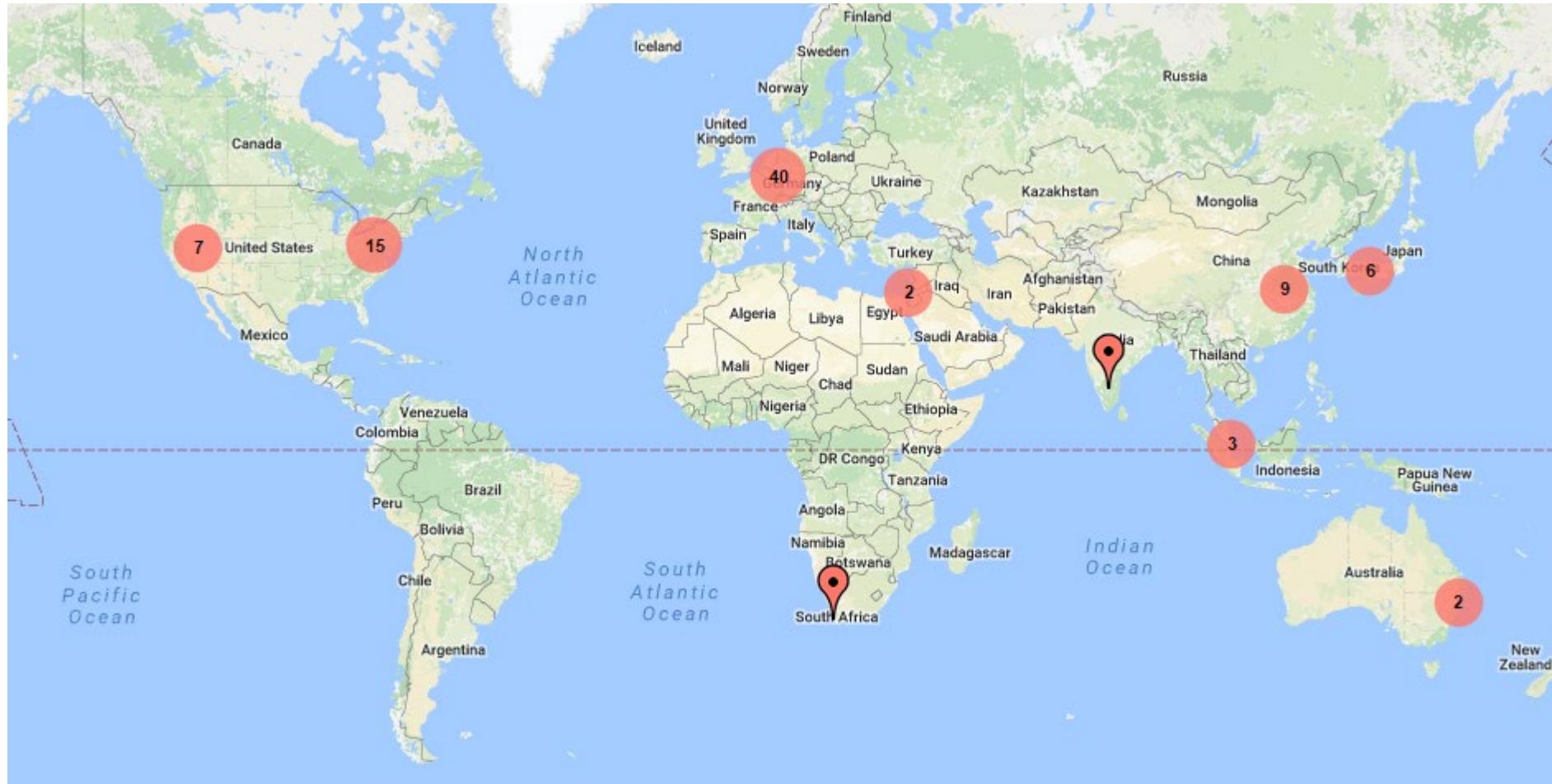
$$P = P_e - P_g$$



Ramsey measurement on Ba^+



The world of traps



References:

Books:

1. Quantum Computation and Quantum Information, Book by Isaac Chuang and Michael Nielsen, Cambridge press
2. Ion traps, book by P. K. Ghosh, Oxford press
3. Principles of ion traps, G. Werth, Springer

Reviews:

1. Leibfried et al. Rev. Mod. Phys. 75, 281 (2003)
2. A. D. Ludlow et al. Rev. Mod. Phys. 87, 637 (2015)

And all the references there in :-)

