

# PAUL TRAP EXPERIMENTS

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## Content

1. Quantum information processing
2. Quantum metrology



# Quantum information processing

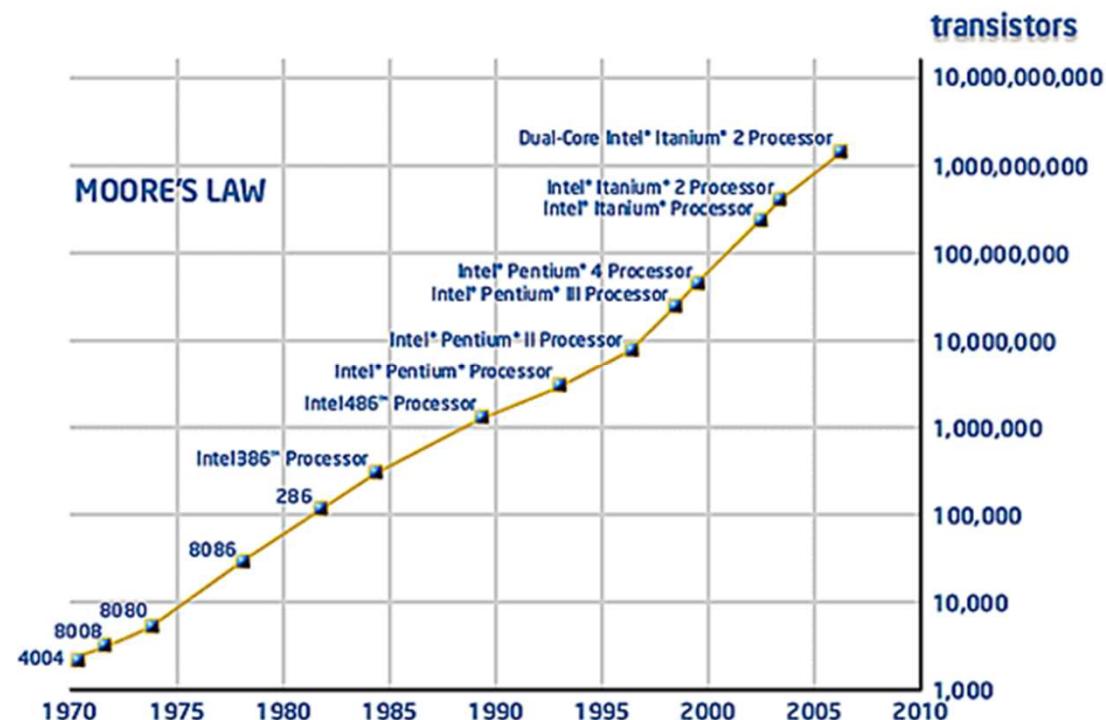
During the past forty years astounding advances have been made in the manufacture of computers. The number of atoms needed to represent a bit in memory has been decreasing exponentially since 1950. Likewise the number of transistors per chip, clock speed, and energy dissipated per logical operation have all followed their own improving exponential trends. This rate of improvement cannot be sustained much longer, at the current rate in the year 2020 one bit of information will require only one atom to represent it. The problem is that at that size the behavior of a computer's components will be dominated by the principles of quantum physics. (Williams & Clearwater)

...(T)he first microprocessor only had 22 hundred transistors. We are looking at something a million times that complex in the next generations-a billion transistors. What that gives us in the way of flexibility to design products is phenomenal."

—Gordon E. Moore (1965)



# Quantum information processing



# Quantum information processing

Classically certain problems are intractable, *for example*:

- Traveling salesman problem
- Prime number factorization problem

Quantum simulation in a classical computer

Can we solve these problems using quantum mechanics?



# Quantum information processing

Are all problems solvable?

Still an unsolved problem in computational mathematics!!!

D. Hilbert in 1928 : Is there an algorithm to solve any decision problem?

A. Church & A. Turing in 1936: No

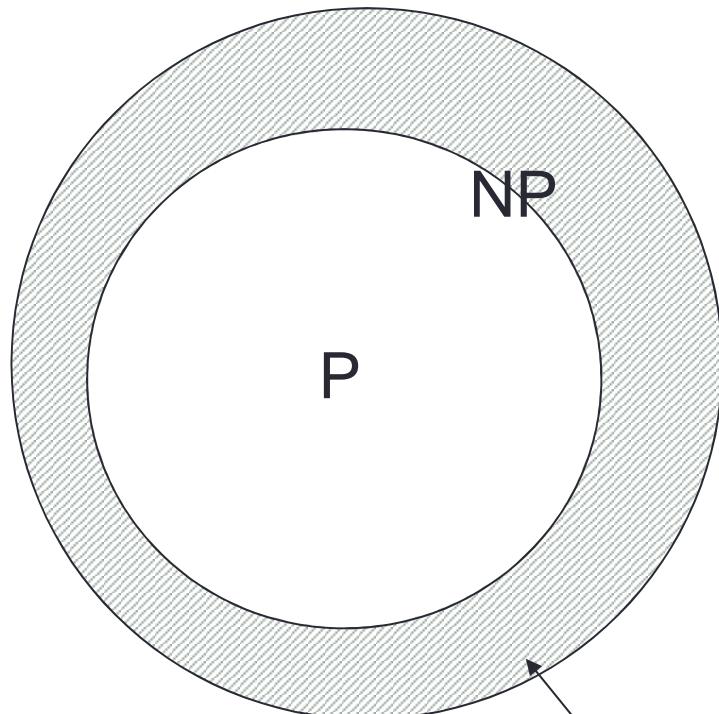
Example of decision problem:

*Goldbach's conjecture*

Every even integer greater than 2 can be written as the sum of two primes



# Quantum information processing



P : Can be solved in Polynomial time  
NP: Solution can be verified in P-time

NP=P ???

NP-complete



# Quantum information processing

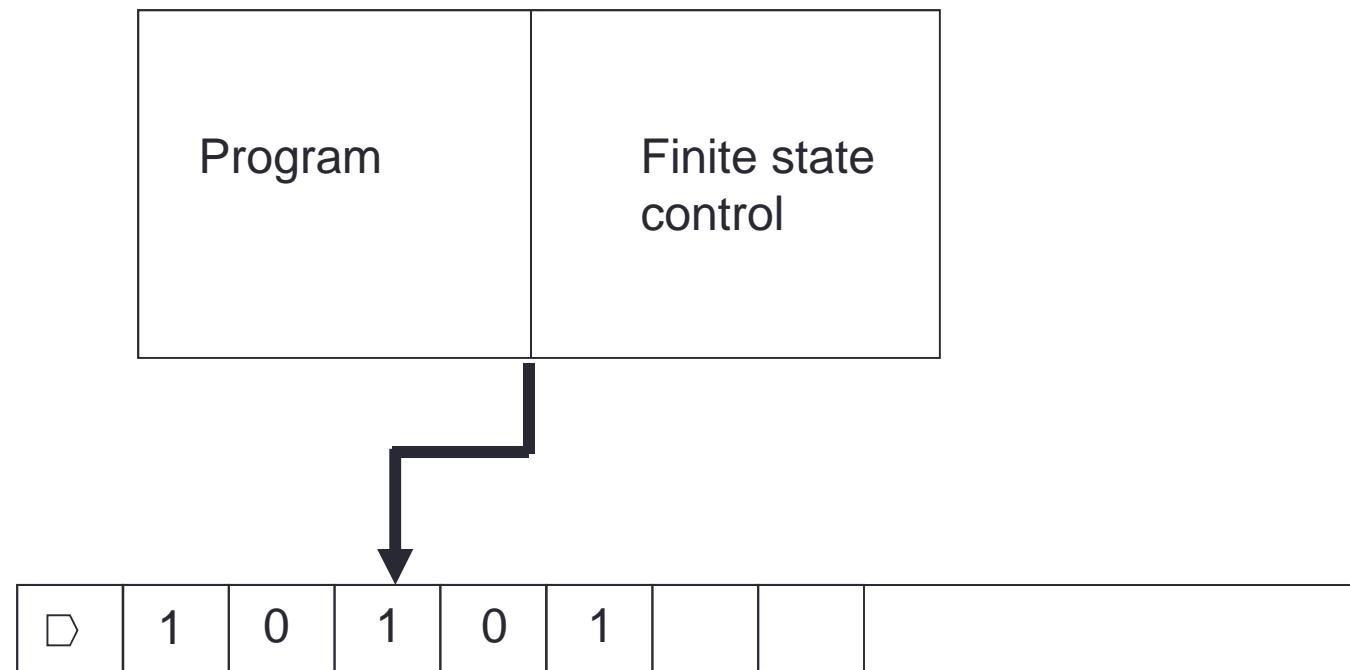
Quantum computation can efficiently solve some of the **so-far known** hard problems in classical computation

BUT

These problems are not known to be NP-complete



# Quantum information processing

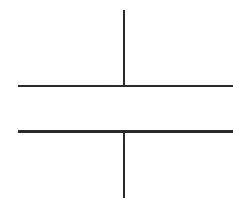


Turing machine & Universal Turing machine

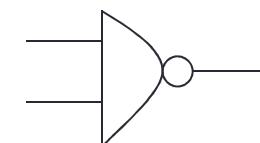


# Quantum information processing

	Classical	Quantum
Memory	BIT	QUBIT
Program	Algorithm	Q-algorithm
Processor	Gates	Q-gates



V=QC



NAND gate



# Quantum information processing

## Quantum unit of information: Qubit

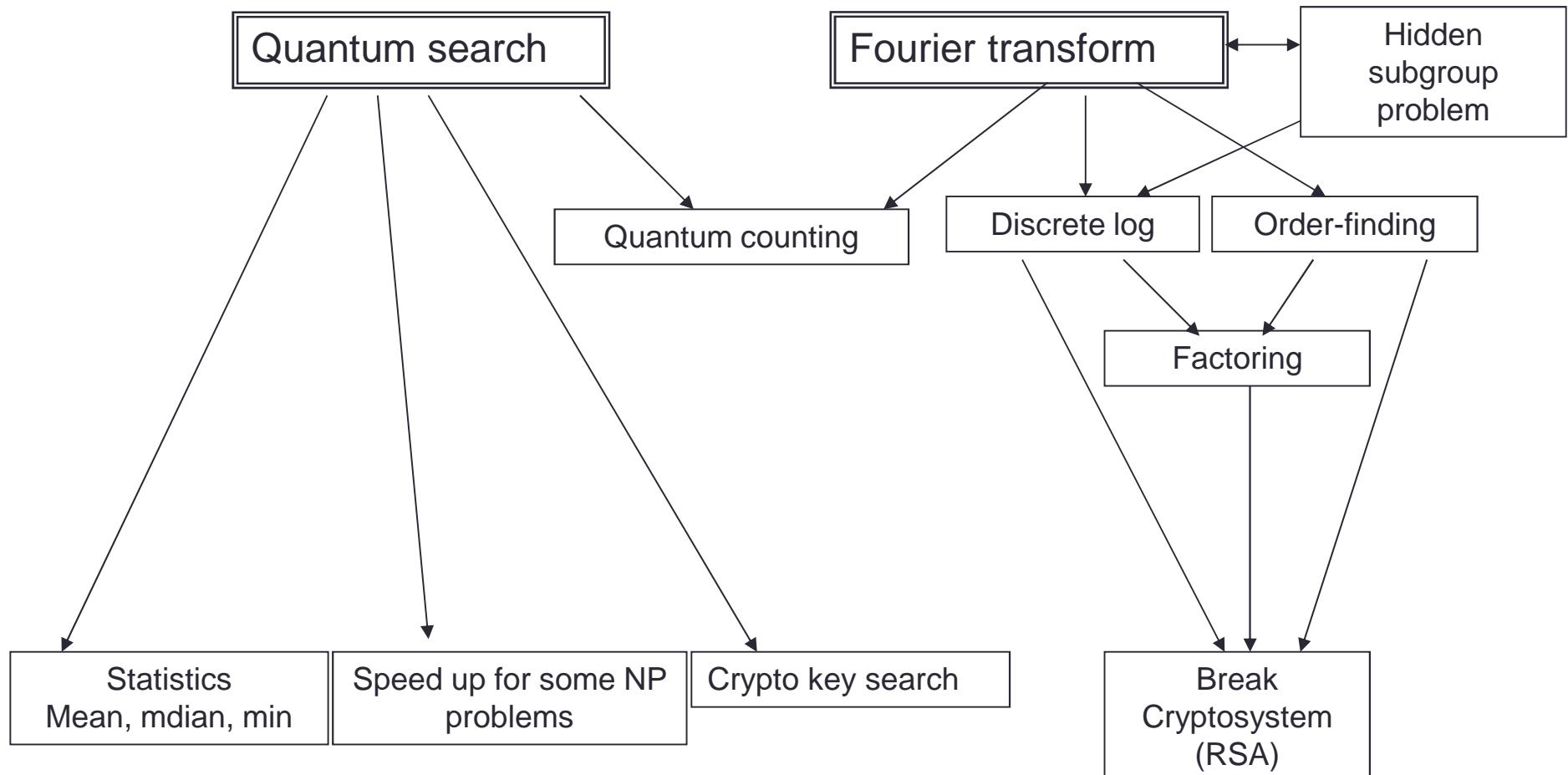
Any two level quantum mechanical system **BUT**

- Possibility to initialize
- Robust against external changes
- Possibility to manipulate using external fields
- Scalable
- Possibility to measure the final state

$$\begin{aligned}|q\rangle &= a|0\rangle + b|1\rangle \\&= e^{i\gamma} \left( \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \right)\end{aligned}$$

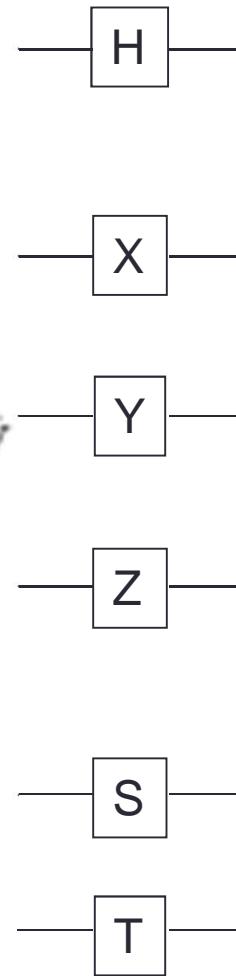
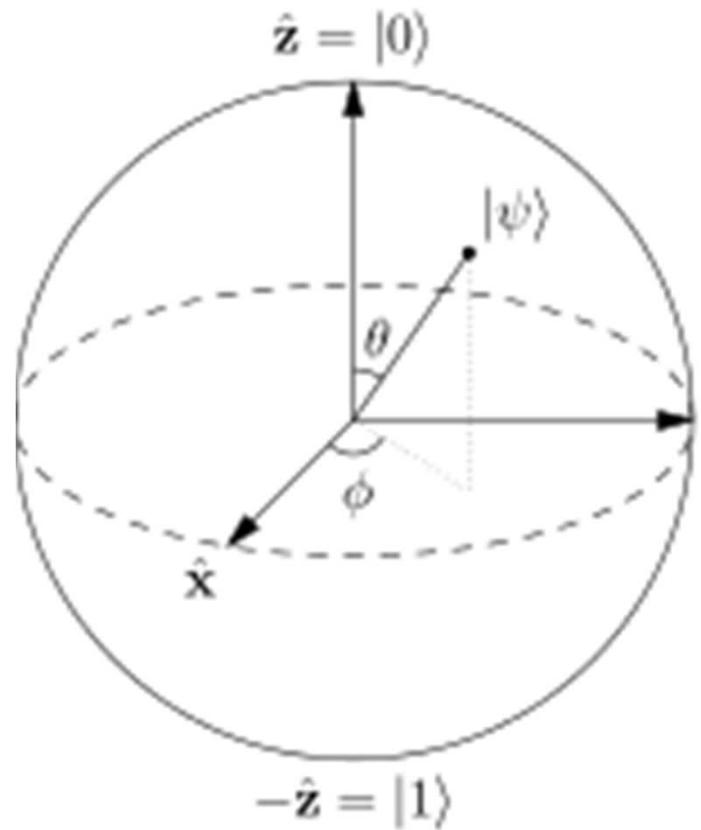


# Quantum information processing



# Quantum information processing

## Single qubit gates



Hadamard

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Pauli-X

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Pauli-Y

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

Pauli-Z

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Phase

$$\begin{bmatrix} 1 & 1 \\ 1 & e^{i\phi} \end{bmatrix}$$

$\pi/8$



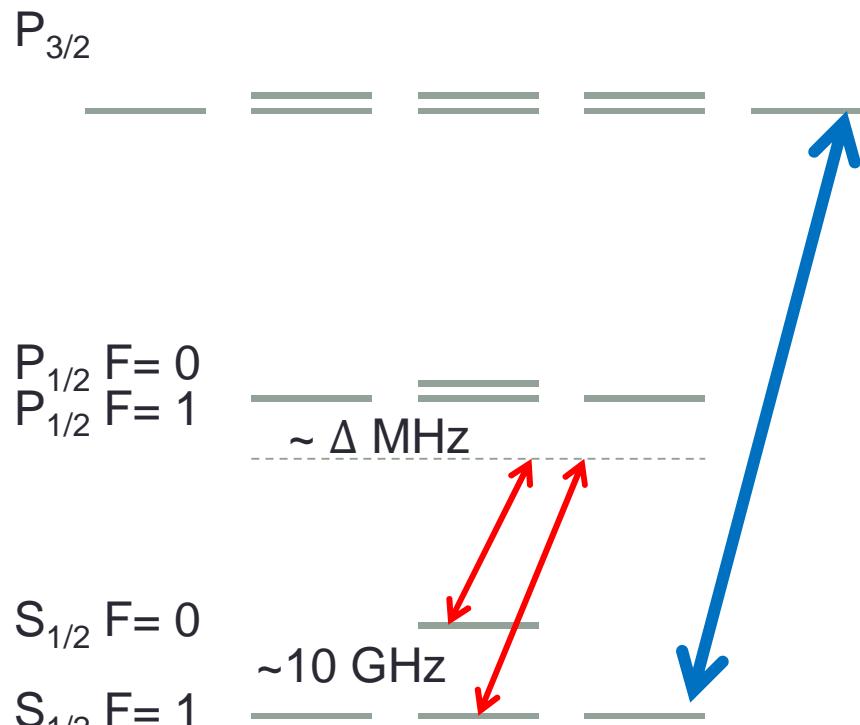
# Quantum information processing

**Universal 2 qubit gate (C)ontrolled-NOT**

Input 2 qubits	Quantum gate	Output 2 qubits
$ q_1\rangle$ $ 00\rangle$ $ 01\rangle$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	$ 00\rangle$ $ q_1\rangle$ $ 01\rangle$
$ q_2\rangle$ $ 10\rangle$ $ 11\rangle$		$ 11\rangle$ $ q_1\rangle \oplus  q_2\rangle$ $ 10\rangle$



# Quantum information processing



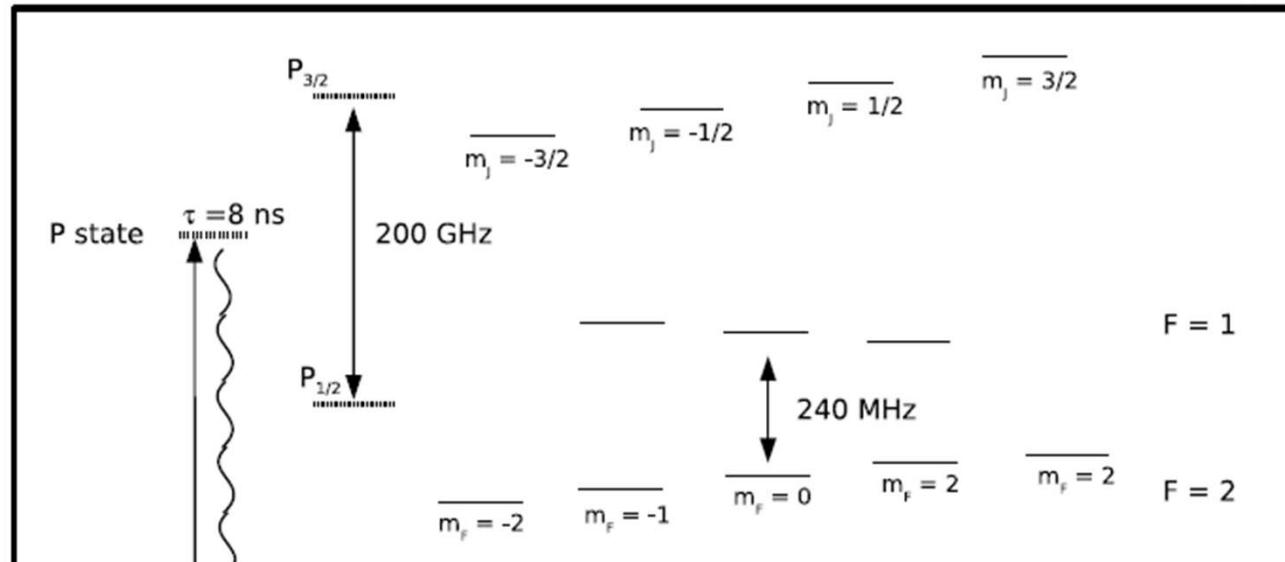
## Hyperfine Qubit:

1. Long lifetime
2. Raman beams / microwaves
3. Odd isotopes of alkaline earth elements
4. Rabi frequency  $\Omega_{RT} = \frac{\Omega_1 \Omega_2}{\Delta}$

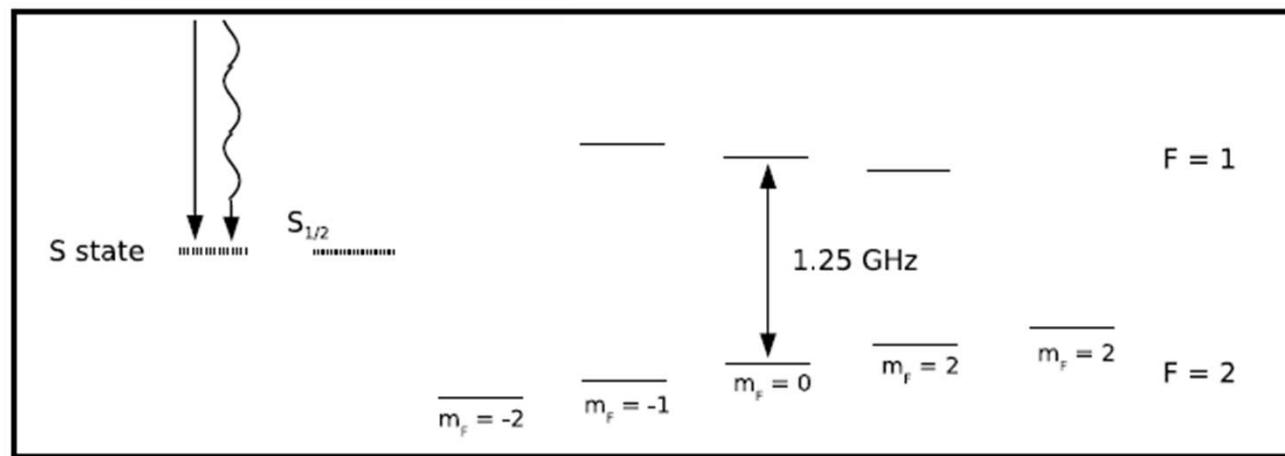


# Quantum information processing

Be-qubit  
NIST

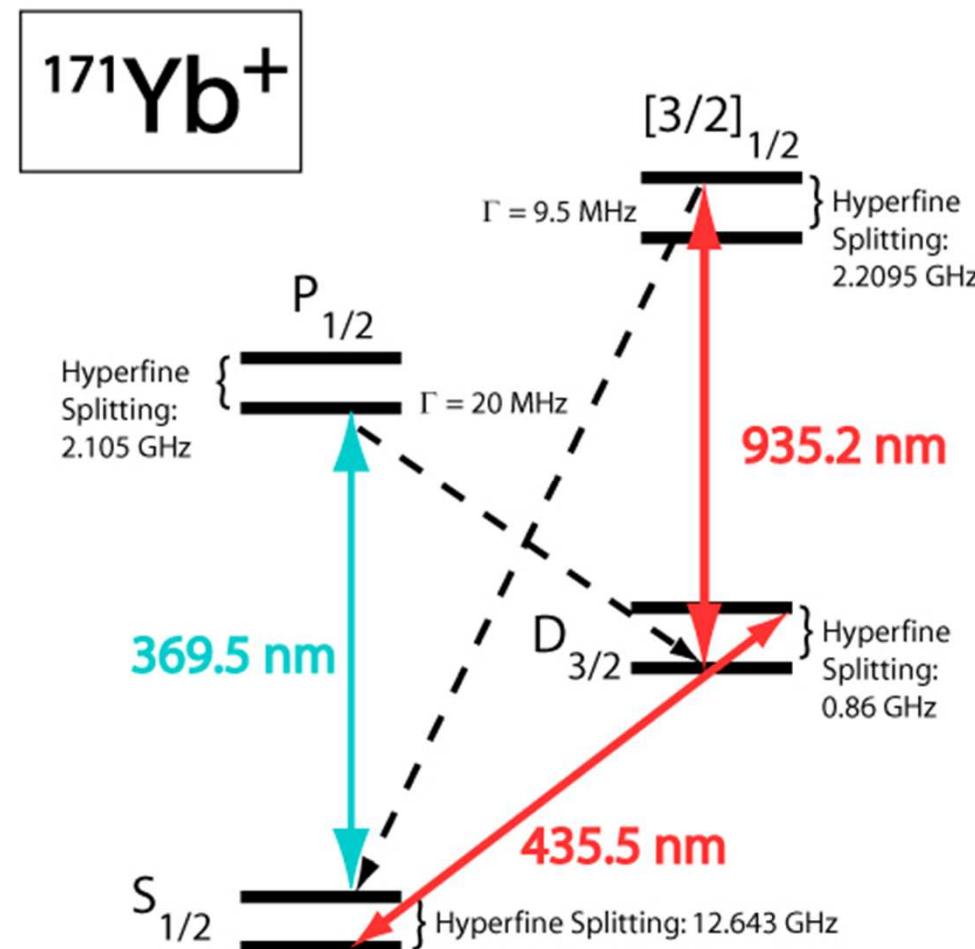


313 nm

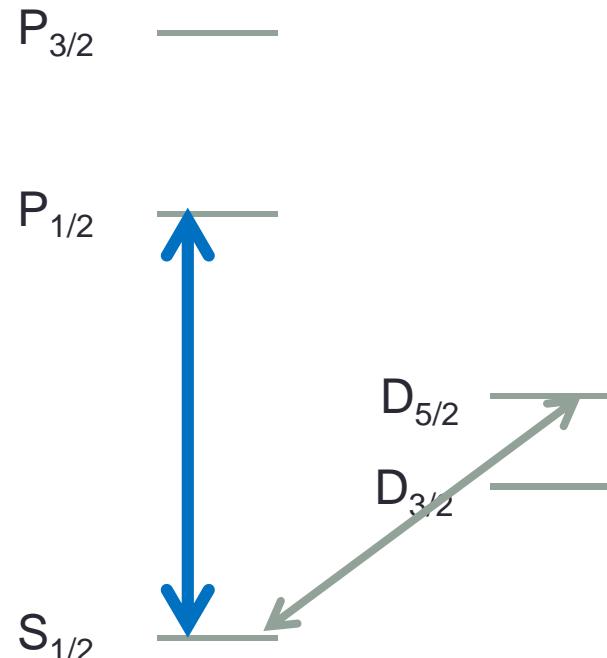


# Quantum information processing

Yb-qubit  
JQI



# Quantum information processing



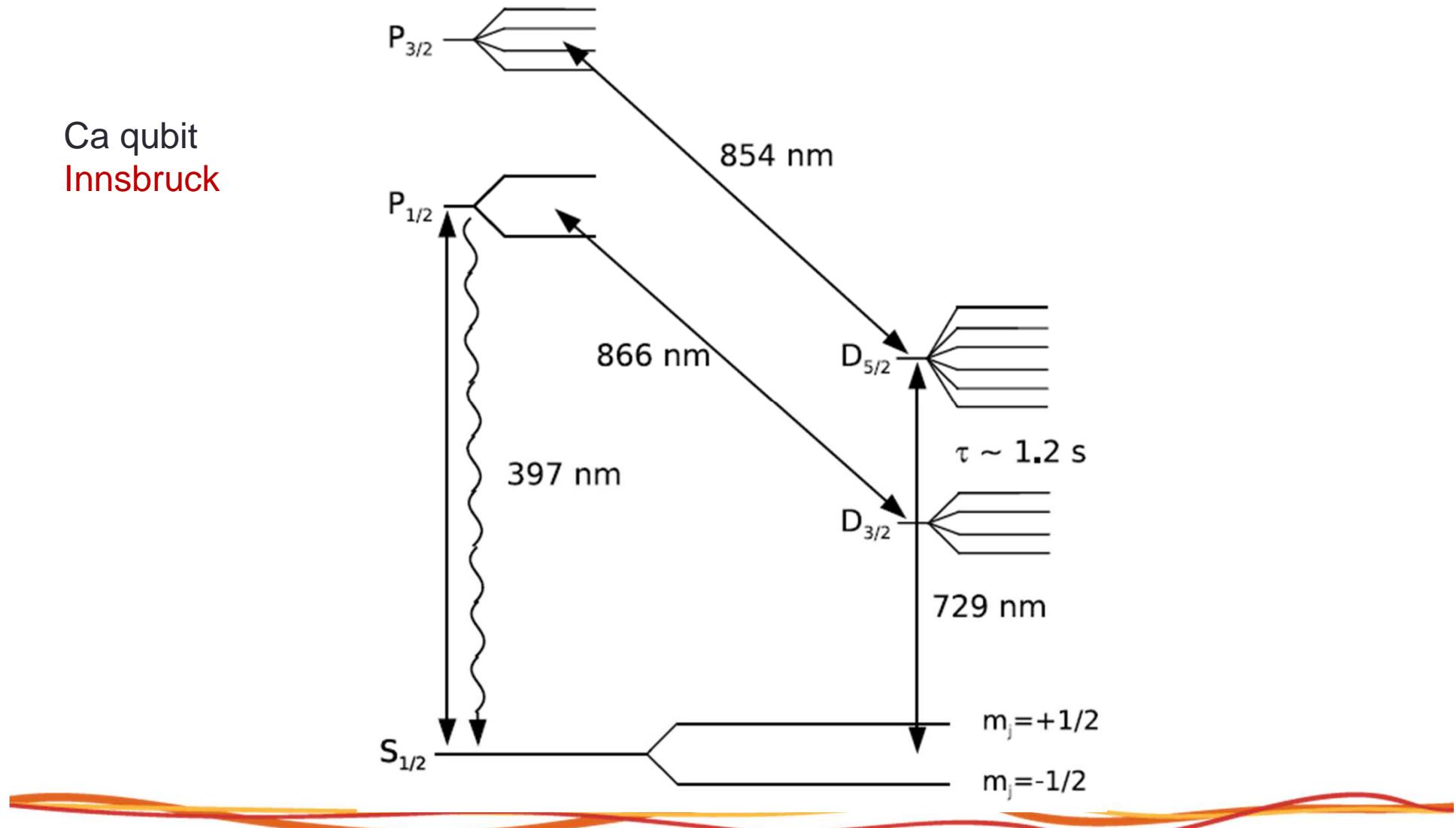
## Optical Qubit:

1. Long lifetime
2. Narrow linewidth lasers
3. even isotopes of alkaline earth elements
4. Rabi frequency  $\Omega_{QT} = \frac{e k}{\hbar} \langle g | |\vec{r}| (\vec{E} \cdot \vec{r}) | e \rangle$

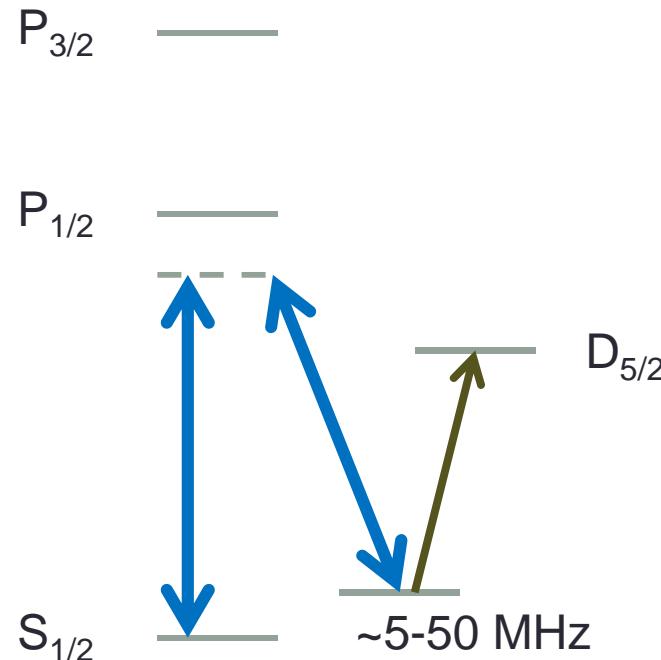


# Quantum information processing

Ca qubit  
Innsbruck



# Quantum information processing



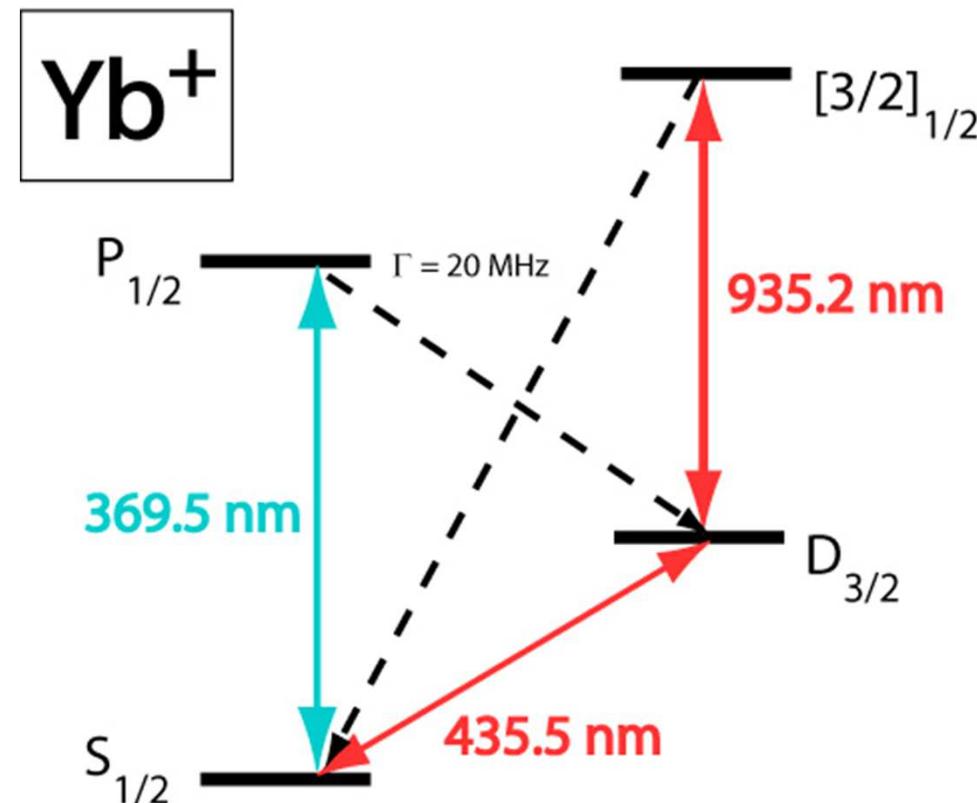
## Zeeman Qubit:

1. Long lifetime
2. Raman lasers / Radio frequency excitation
3. Even/odd isotopes of alkaline earth elements
4. Susceptible to magnetic field noise



# Quantum information processing

Yb qubit  
Ulm University



# Choice of qubit

1																			18
1 <b>H</b> 1.008	2																	2 <b>He</b> 4.0026	
3 <b>Li</b> 6.94	4 <b>Be</b> 9.0122																	10 <b>Ne</b> 20.180	
11 <b>Na</b> 22.990	12 <b>Mg</b> 24.305	3	4	5	6	7	8	9	10	11	12	13 <b>B</b> 10.81	14 <b>C</b> 12.011	15 <b>N</b> 14.007	16 <b>O</b> 15.999	17 <b>F</b> 18.998		18 <b>Ar</b> 39.948	
19 <b>K</b> 39.098	20 <b>Ca</b> 40.078	21 <b>Sc</b> 44.956	22 <b>Ti</b> 47.867	23 <b>V</b> 50.942	24 <b>Cr</b> 51.996	25 <b>Mn</b> 54.938	26 <b>Fe</b> 55.845	27 <b>Co</b> 58.933	28 <b>Ni</b> 58.693	29 <b>Cu</b> 63.546	30 <b>Zn</b> 65.38	31 <b>Ga</b> 69.723	32 <b>Ge</b> 72.630	33 <b>As</b> 74.922	34 <b>Se</b> 78.97	35 <b>Br</b> 79.904	36 <b>Kr</b> 83.798		
37 <b>Rb</b> 85.468	38 <b>Sr</b> 87.62	39 <b>Y</b> 88.906	40 <b>Zr</b> 91.224	41 <b>Nb</b> 92.906	42 <b>Mo</b> 95.95	43 <b>Tc</b> (98)	44 <b>Ru</b> 101.07	45 <b>Rh</b> 102.91	46 <b>Pd</b> 106.42	47 <b>Ag</b> 107.87	48 <b>Cd</b> 112.41	49 <b>In</b> 114.82	50 <b>Sn</b> 118.71	51 <b>Sb</b> 121.76	52 <b>Te</b> 127.60	53 <b>I</b> 126.90	54 <b>Xe</b> 131.29		
55 <b>Cs</b> 132.91	56 <b>Ba</b> 137.33	57-71 *	72 <b>Hf</b> 178.49	73 <b>Ta</b> 180.95	74 <b>W</b> 183.84	75 <b>Re</b> 186.21	76 <b>Os</b> 190.23	77 <b>Ir</b> 192.22	78 <b>Pt</b> 195.08	79 <b>Au</b> 196.97	80 <b>Hg</b> 200.59	81 <b>Tl</b> 204.38	82 <b>Pb</b> 207.2	83 <b>Bi</b> 208.98	84 <b>Po</b> (209)	85 <b>At</b> (210)	86 <b>Rn</b> (222)		
87 <b>Fr</b> (223)	88 <b>Ra</b> (226)	89-103 #	104 <b>Rf</b> (265)	105 <b>Db</b> (268)	106 <b>Sg</b> (271)	107 <b>Bh</b> (270)	108 <b>Hs</b> (277)	109 <b>Mt</b> (276)	110 <b>Ds</b> (281)	111 <b>Rg</b> (280)	112 <b>Cn</b> (285)	113 <b>Nh</b> (286)	114 <b>Fl</b> (289)	115 <b>Mc</b> (289)	116 <b>Lv</b> (293)	117 <b>Ts</b> (294)	118 <b>Og</b> (294)		

\* Lanthanide series

57 <b>La</b> 138.91	58 <b>Ce</b> 140.12	59 <b>Pr</b> 140.91	60 <b>Nd</b> 144.24	61 <b>Pm</b> (145)	62 <b>Sm</b> 150.36	63 <b>Eu</b> 151.96	64 <b>Gd</b> 157.25	65 <b>Tb</b> 158.93	66 <b>Dy</b> 162.50	67 <b>Ho</b> 164.93	68 <b>Er</b> 167.26	69 <b>Tm</b> 168.93	70 <b>Yb</b> 173.05	71 <b>Lu</b> 174.97
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# Actinide series

89 <b>Ac</b> (227)	90 <b>Th</b> 232.04	91 <b>Pa</b> 231.04	92 <b>U</b> 238.03	93 <b>Np</b> (237)	94 <b>Pu</b> (244)	95 <b>Am</b> (243)	96 <b>Cm</b> (247)	97 <b>Bk</b> (247)	98 <b>Cf</b> (251)	99 <b>Es</b> (252)	100 <b>Fm</b> (257)	101 <b>Md</b> (258)	102 <b>No</b> (259)	103 <b>Lr</b> (262)
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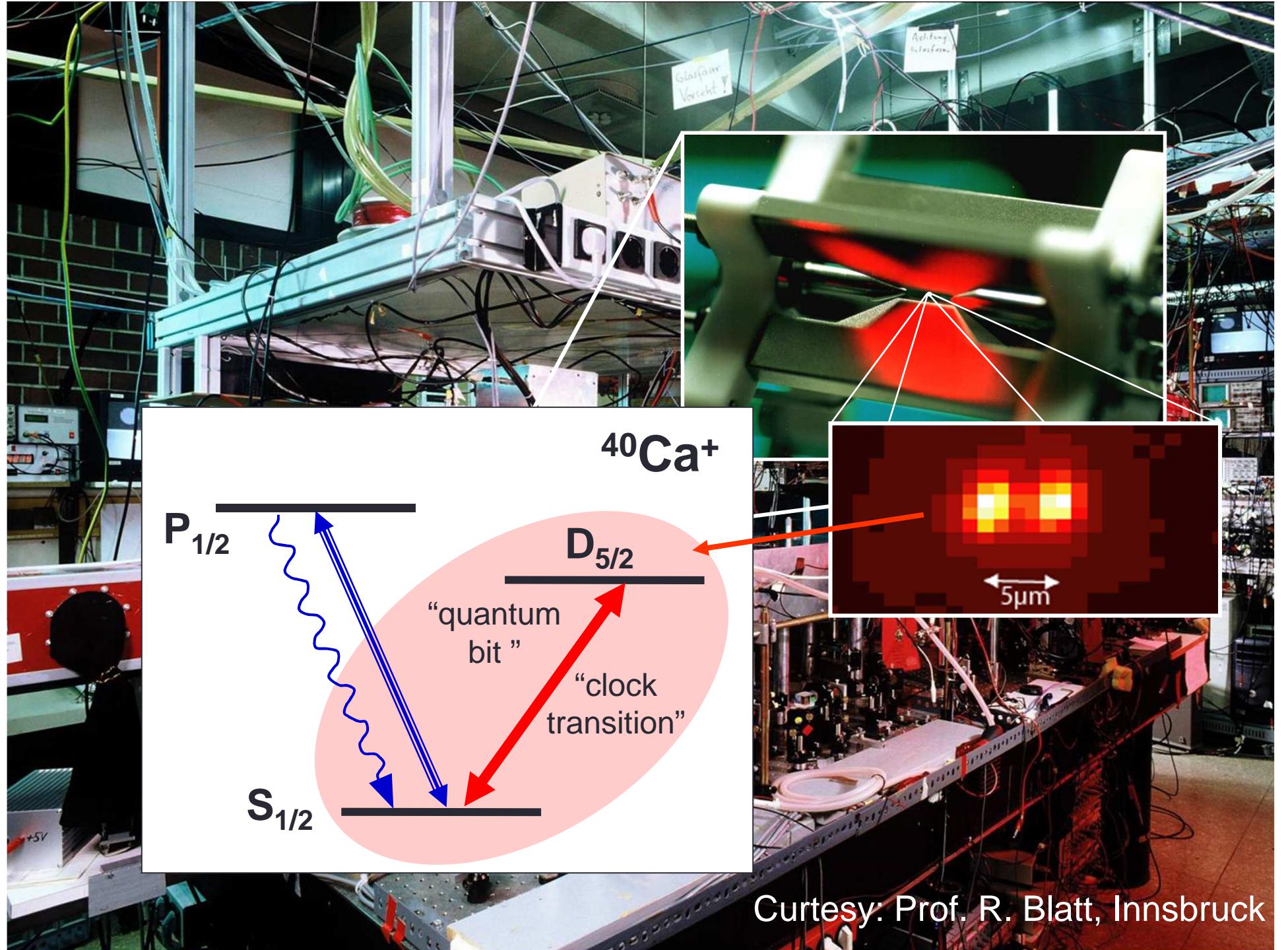


# Choice of qubit

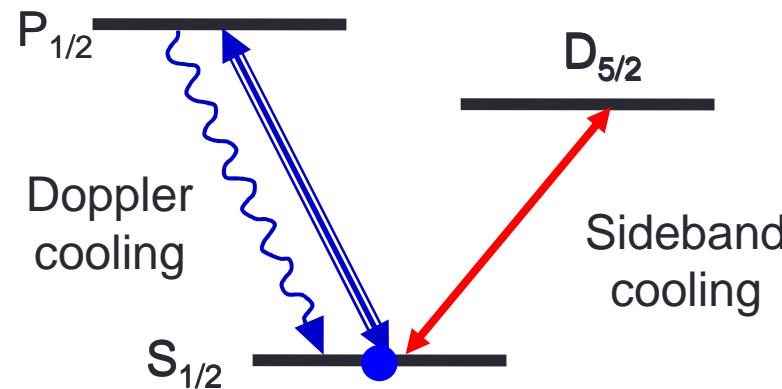
1. The relevant transitions should have accessible laser wavelength
2. Light atoms have higher Lamb-Dicke parameter
3. Suitable qubit transition

$$\eta = k \sqrt{\frac{\hbar}{2m\nu_{sec}}} = \frac{2\pi}{\lambda} \sqrt{\frac{\hbar}{2m\nu_{sec}}}$$





# Quantum information processing

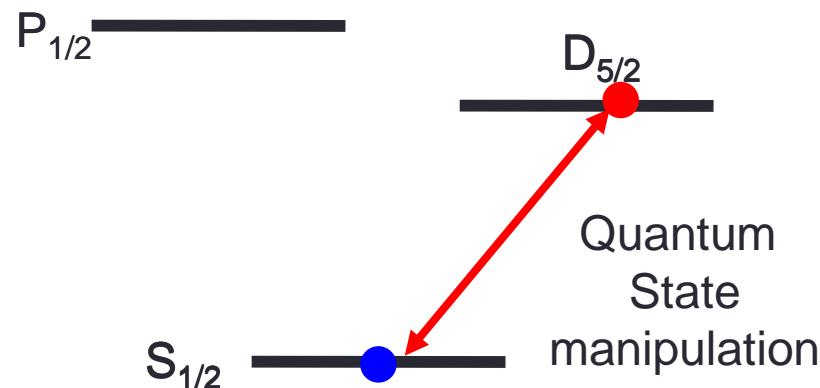


## The recipe

- Initialization
- Quantum state manipulation
- Quantum state measurement

# Quantum information processing

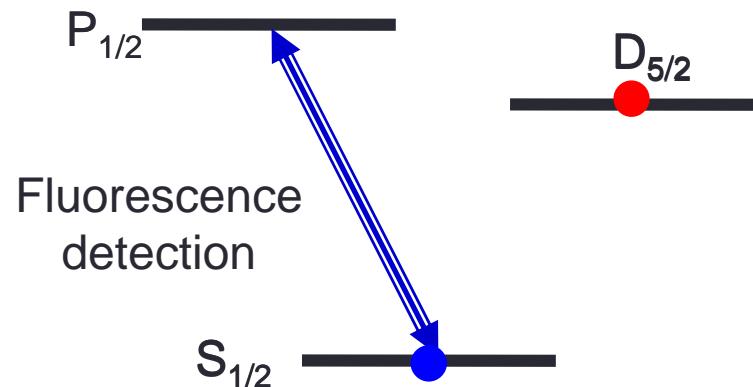
## The recipe



- Initialization
- Quantum state manipulation
- Quantum state measurement

# Quantum information processing

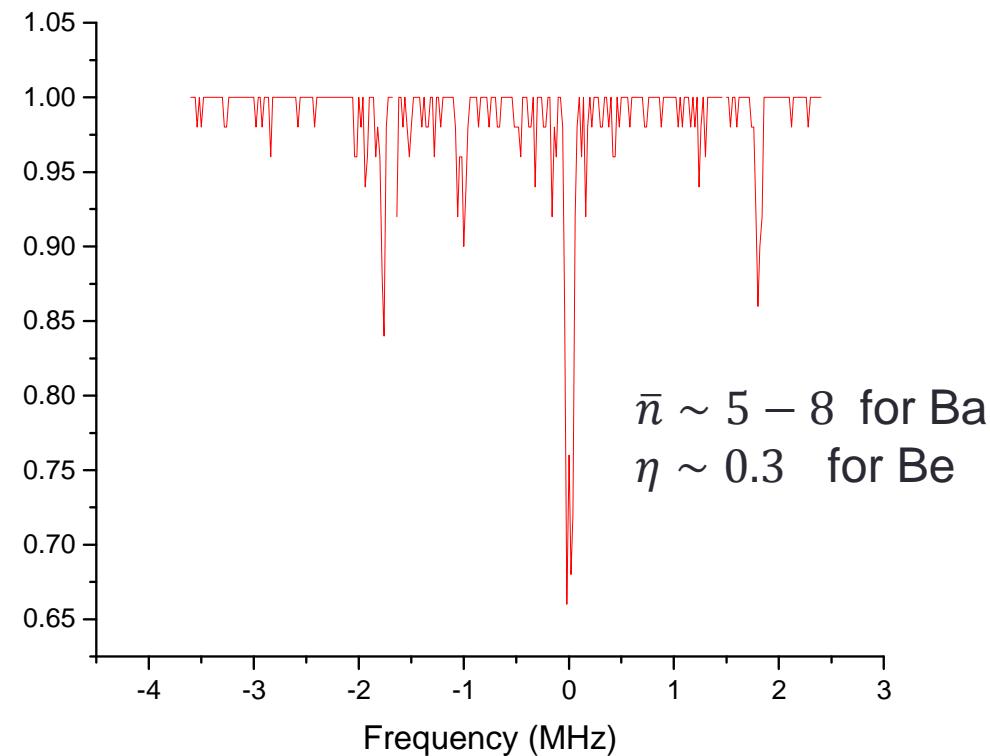
## The recipe



- Initialization
- Quantum state manipulation
- Quantum state measurement

# Quantum information processing

## Initialization



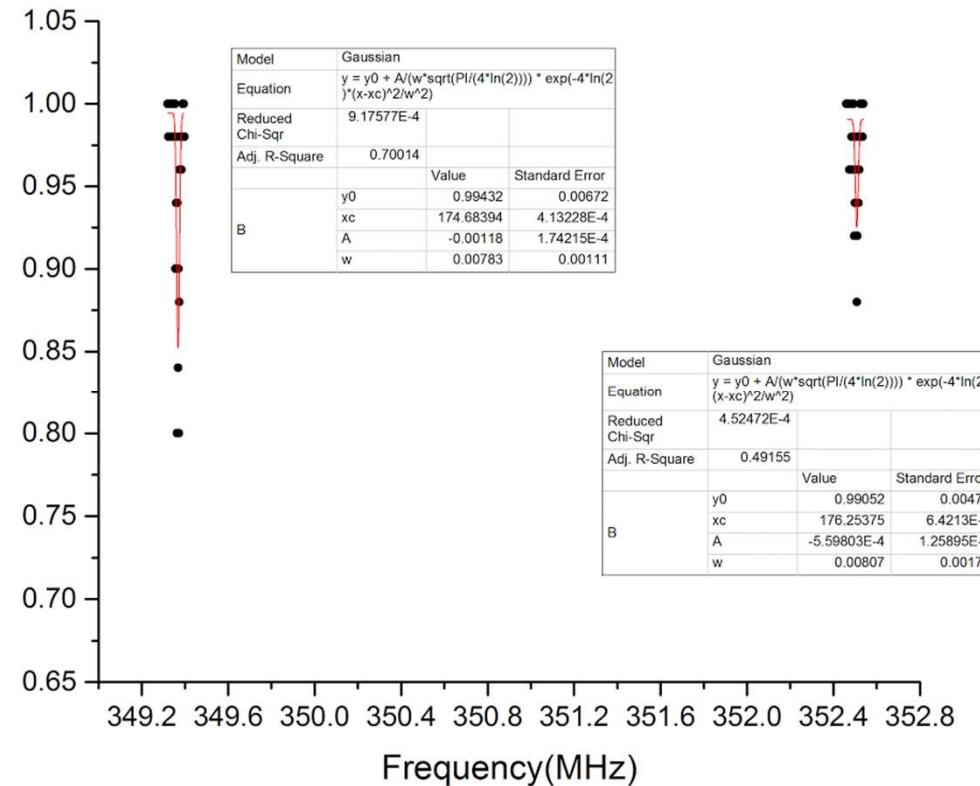
Step1: Doppler cooling 0.5-1 ms  
Goal: reach  $\eta \ll 1$

$T = 1 \text{ ms}$



# Quantum information processing

## Initialization



Step2: Side-band cooling 1-5 ms  
 Goal: reach  $\bar{n} \sim 0.1$

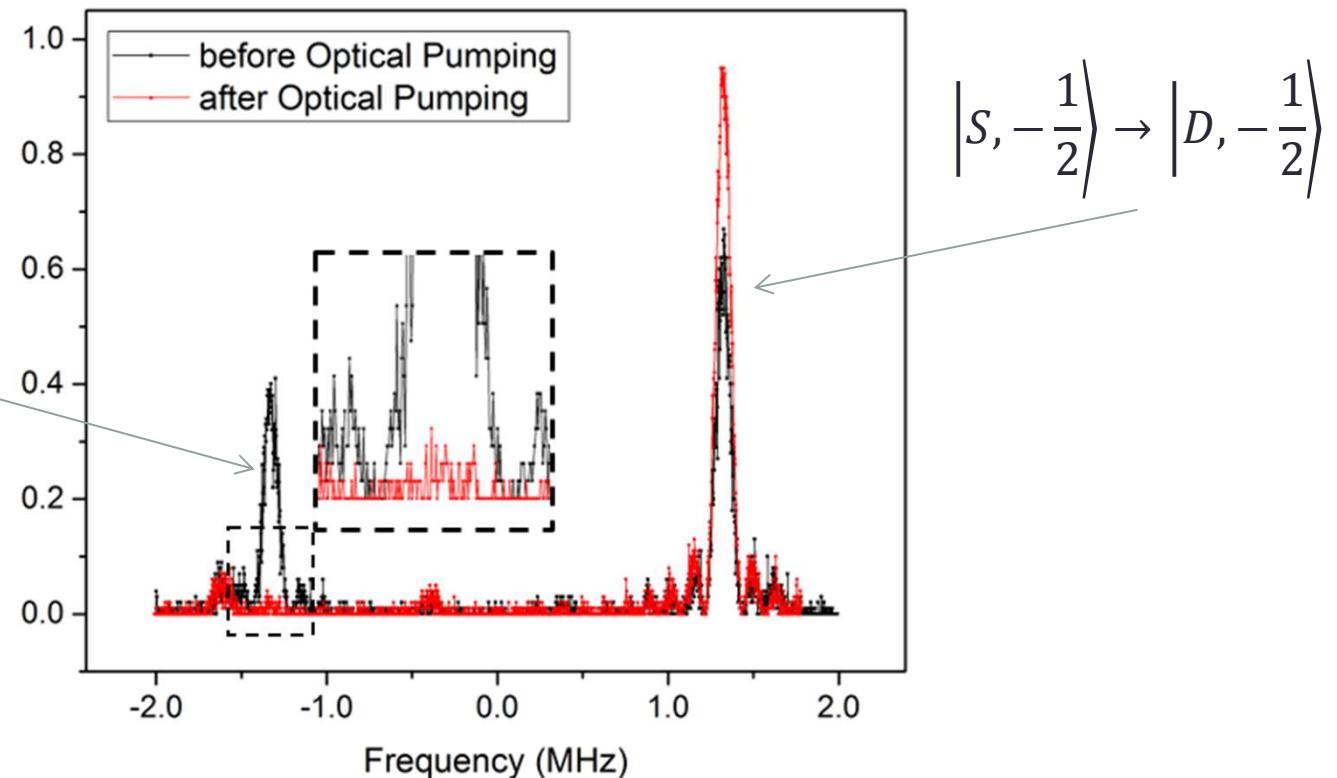
T = 1+5 ms



# Quantum information processing

## Initialization

$$\left|S, +\frac{1}{2}\right\rangle \rightarrow \left|D, +\frac{1}{2}\right\rangle$$



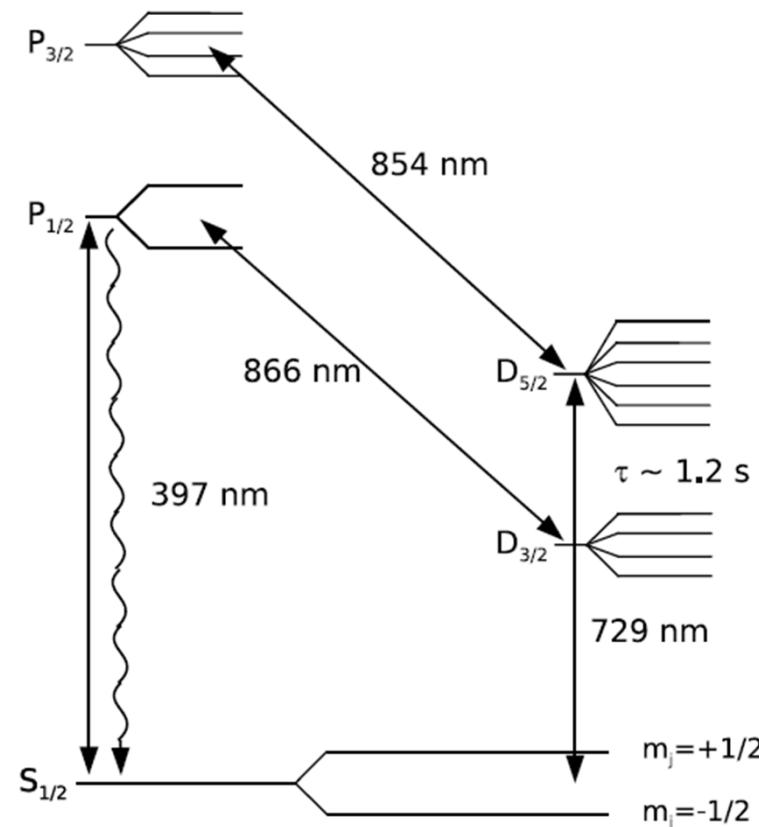
Step2: Optical pumping 10 – 100  $\mu$ s  
Goal: population in  $|g, 0\rangle \geq 99.99\%$

T = 1+5+0.1 ms



# Quantum information processing

## Initialization



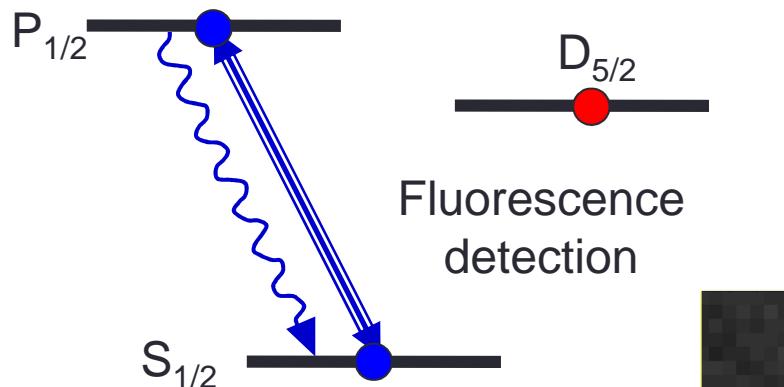
Step2: Optical pumping 10 – 100  $\mu$ s  
Goal: population in  $|g, 0\rangle \geq 99.99\%$

$T = 1+5+0.1$  ms



# Quantum information processing

## Measurement



Two ions:

Spatially resolved  
detection with  
CCD camera

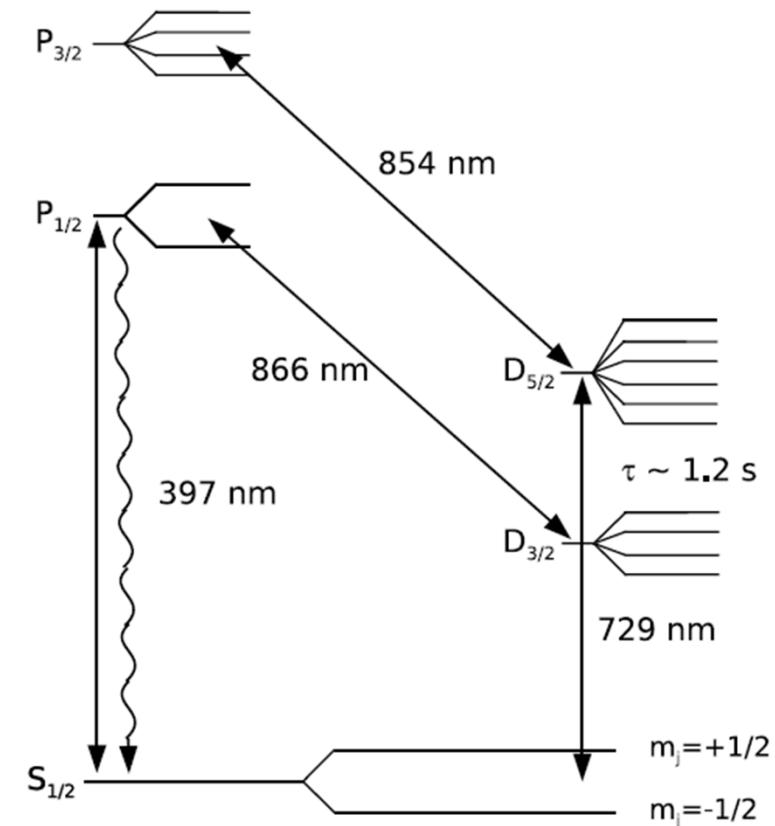
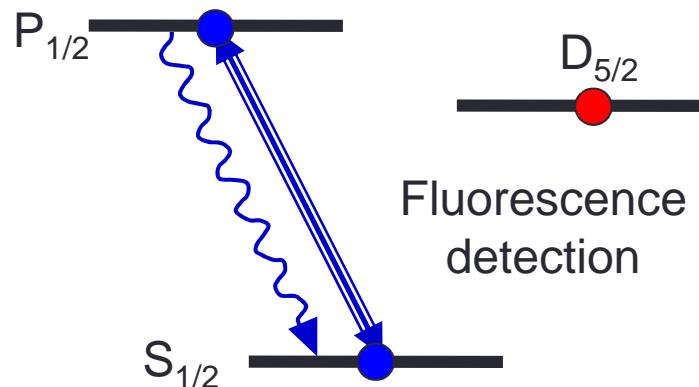
50 experiments / s

Repeat experiments  
100-200 times



# Quantum information processing

## Measurement / state detection

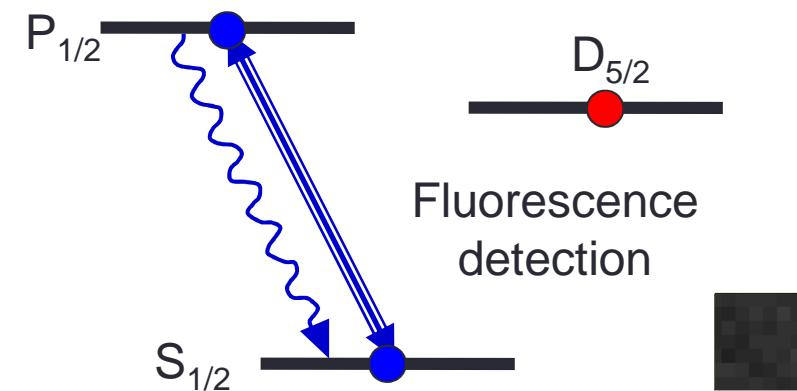
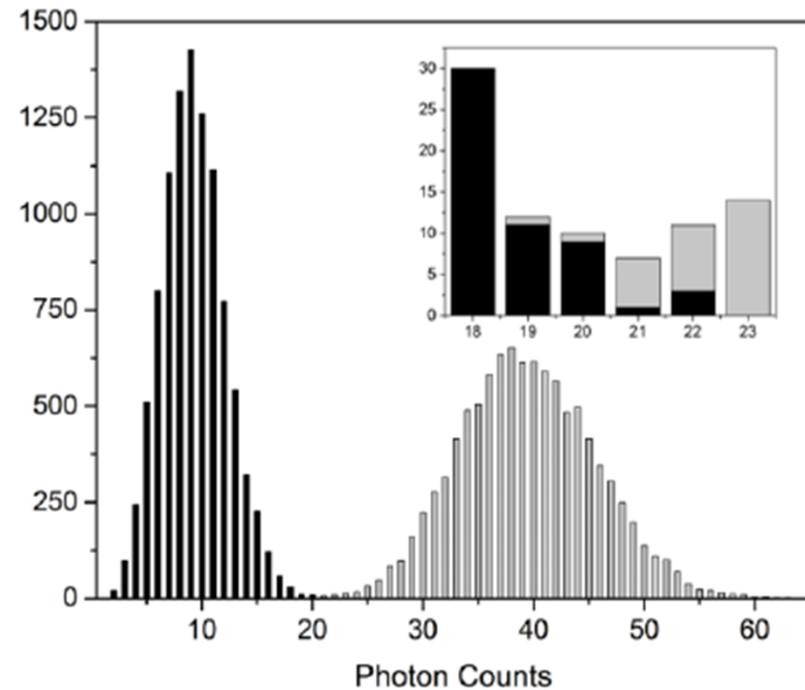


Problem 10: Considering the optical dipole transition in Ca ion, calculate the number of photons arriving the CCD camera with an overall collection efficiency of 0.1%



# Quantum information processing

## Measurement / state discrimination



Typical error in state discrimination < 0.1% within a time of 1 ms

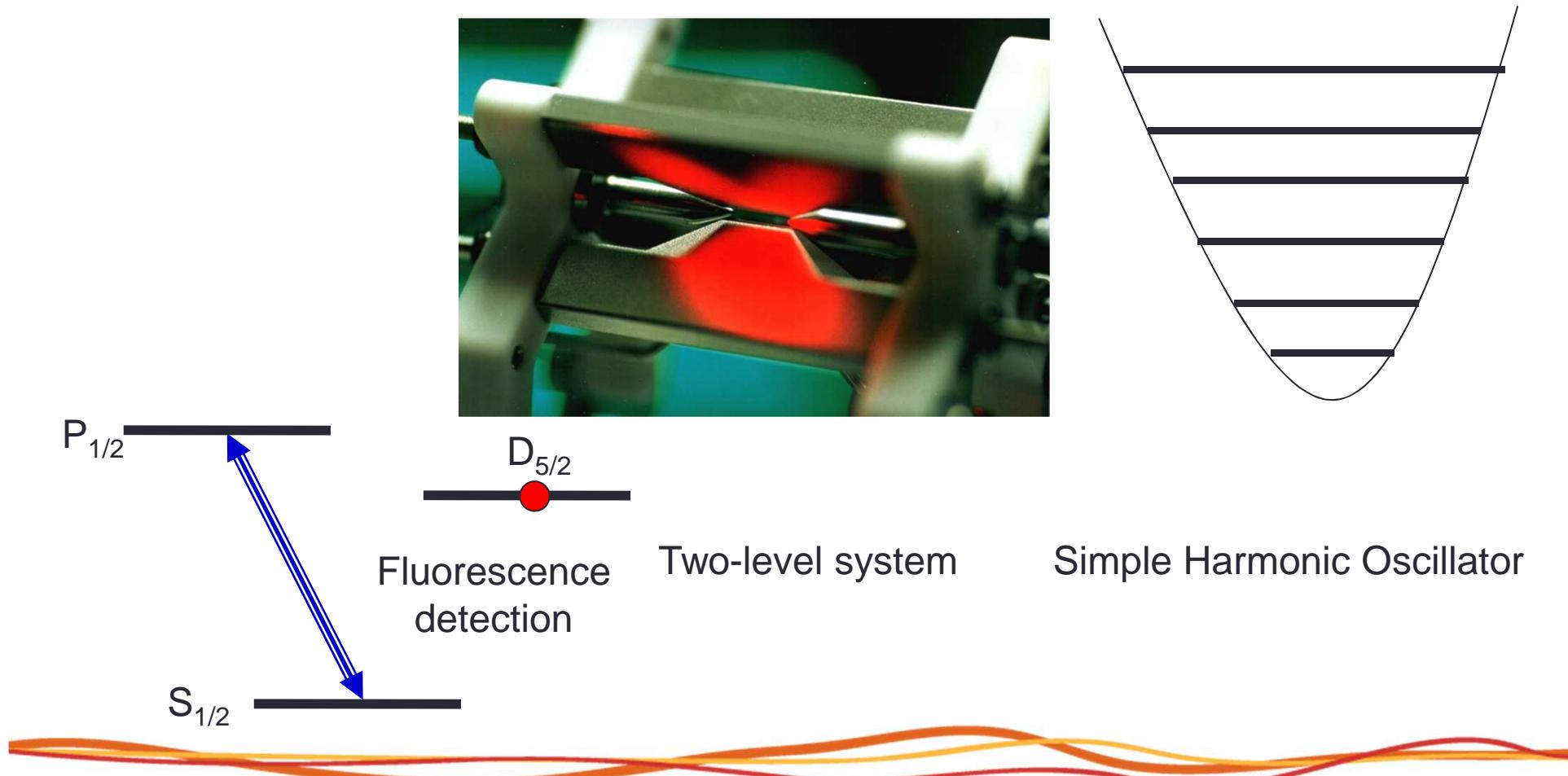
$$T = 1+5+0.1+1 \text{ ms}$$



# Quantum information processing

## Q-state manipulation

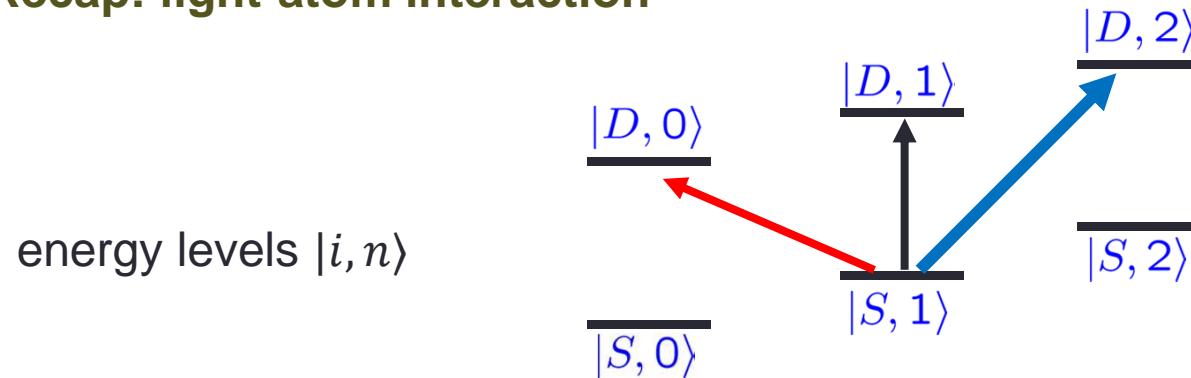
Summarize the model:



# Quantum information processing

## Q-state manipulation

**Recap: light-atom interaction**



$$1. \delta = 0, H_{car} = (h / 4 * \pi) \Omega_0 (\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi})$$

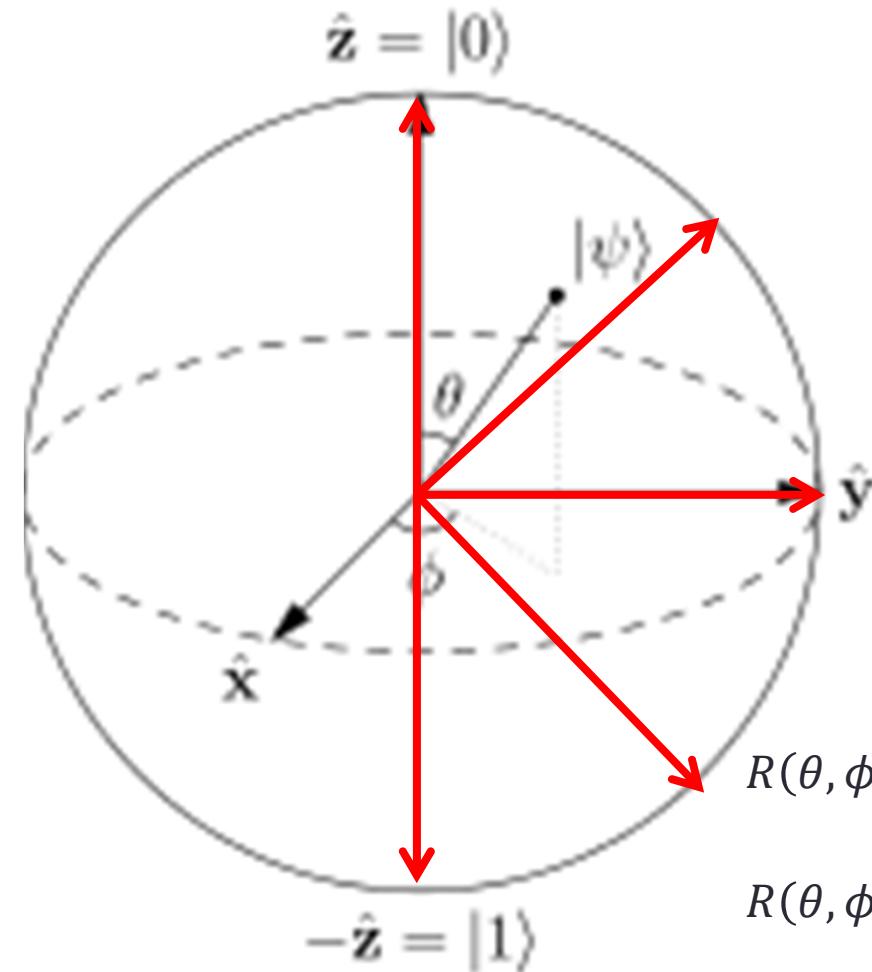
$$2. \delta = -\nu, H_{rsb} = (h / 4 * \pi) \Omega_0 \eta (a \sigma_+ e^{i\phi} + a^\dagger \sigma_- e^{-i\phi})$$

$$3. \delta = \nu, H_{bsb} = (h / 4 * \pi) \Omega_0 \eta (a^\dagger \sigma_+ e^{i\phi} + a \sigma_- e^{-i\phi})$$



# Quantum information processing

## Q-state manipulation – single qubit



$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

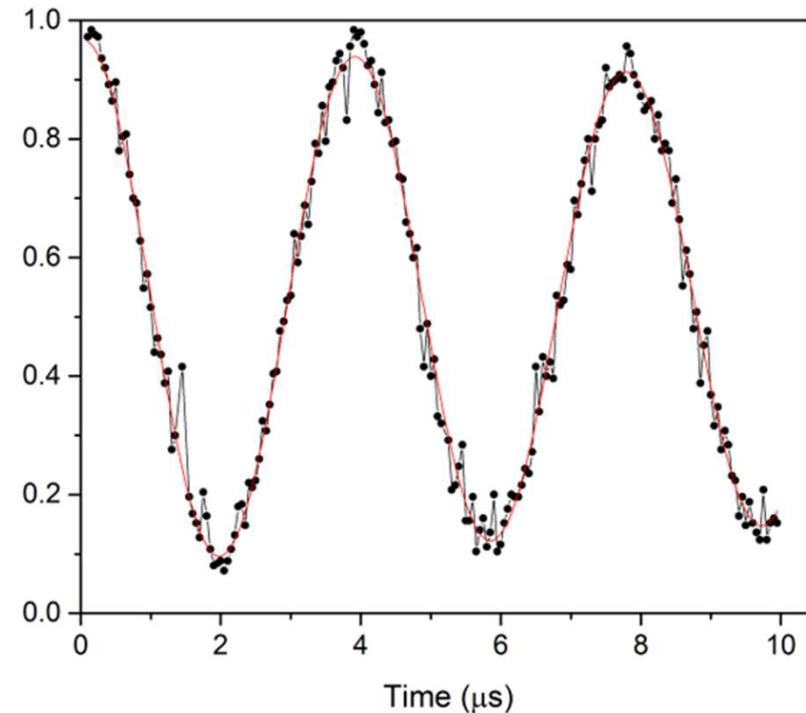
$$R(\theta, \phi) = \exp\left(\frac{i\theta}{2}(e^{i\phi}\sigma_+ + e^{-i\phi}\sigma_-)\right)$$

$$R(\theta, \phi) = I \cos\left(\frac{\theta}{2}\right) + i(\sigma_x \cos(\phi) - \sigma_y \sin(\phi)) \sin\left(\frac{\theta}{2}\right)$$



# Quantum information processing

## Q-state manipulation – single qubit



$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$R(\theta, \phi) = \exp\left(\frac{i\theta}{2}(e^{i\phi} \sigma_+ + e^{-i\phi} \sigma_-)\right)$$

$$R(\theta, \phi) = I \cos\left(\frac{\theta}{2}\right) + i(\sigma_x \cos(\phi) - \sigma_y \sin(\phi)) \sin\left(\frac{\theta}{2}\right)$$



# Generation of Bell state

⋮  
 $|DD1\rangle$  ———  
 $|DD0\rangle$  ———

**Generation of Bell states:**  $|SD\rangle + |DS\rangle$   
**Pulse sequence:**

⋮ ⋮  
 $|DS1\rangle$  ——— ———  $|SD1\rangle$   
 $|DS0\rangle$  ——— ———  $|SD0\rangle$

⋮  
 $|SS1\rangle$  ———  
 $|SS0\rangle$  ——— ●

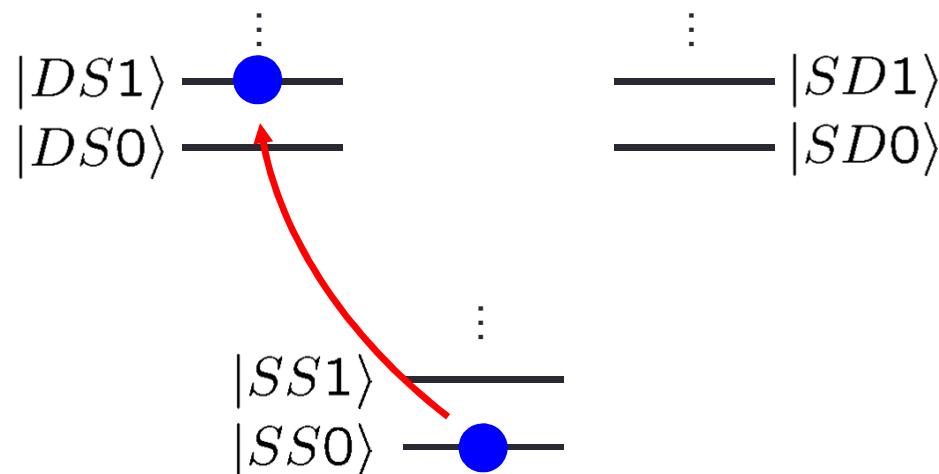
$|SS0\rangle$



# Generation of Bell state

$|DD1\rangle$      $\vdots$   
 $|DD0\rangle$

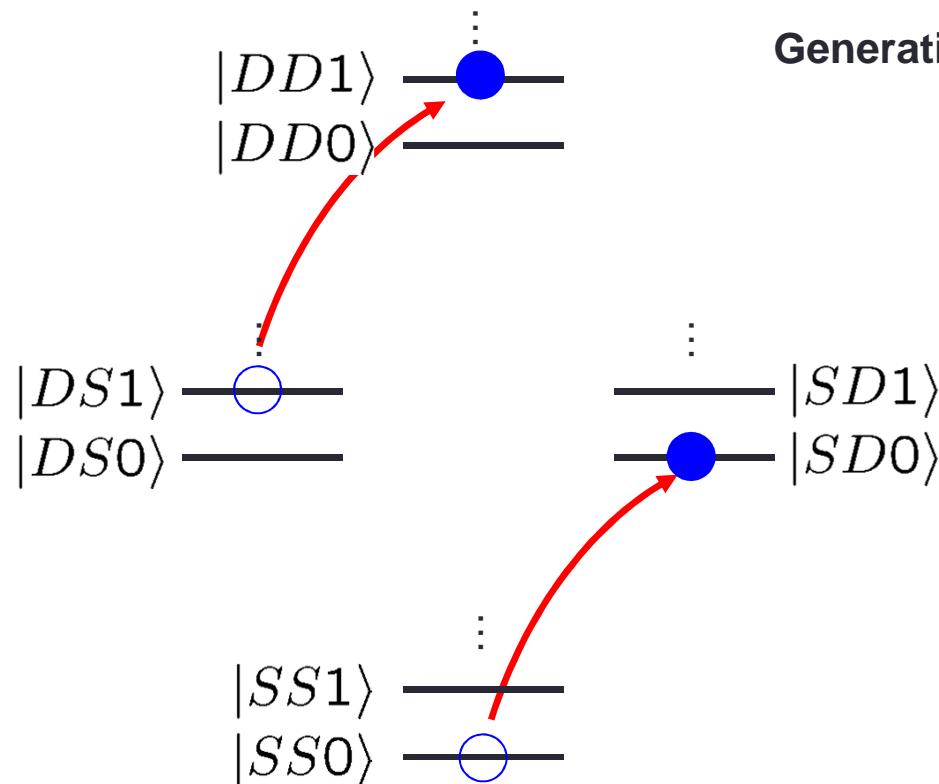
**Generation of Bell states:**  $|SD\rangle + |DS\rangle$   
**Pulse sequence:**  
**Ion 1:**  $\pi/2$ , blue sideband



$|SS0\rangle + |DS1\rangle$



# Generation of Bell state



**Generation of Bell states:**  $|SD> + |DS>$

Pulse sequence:

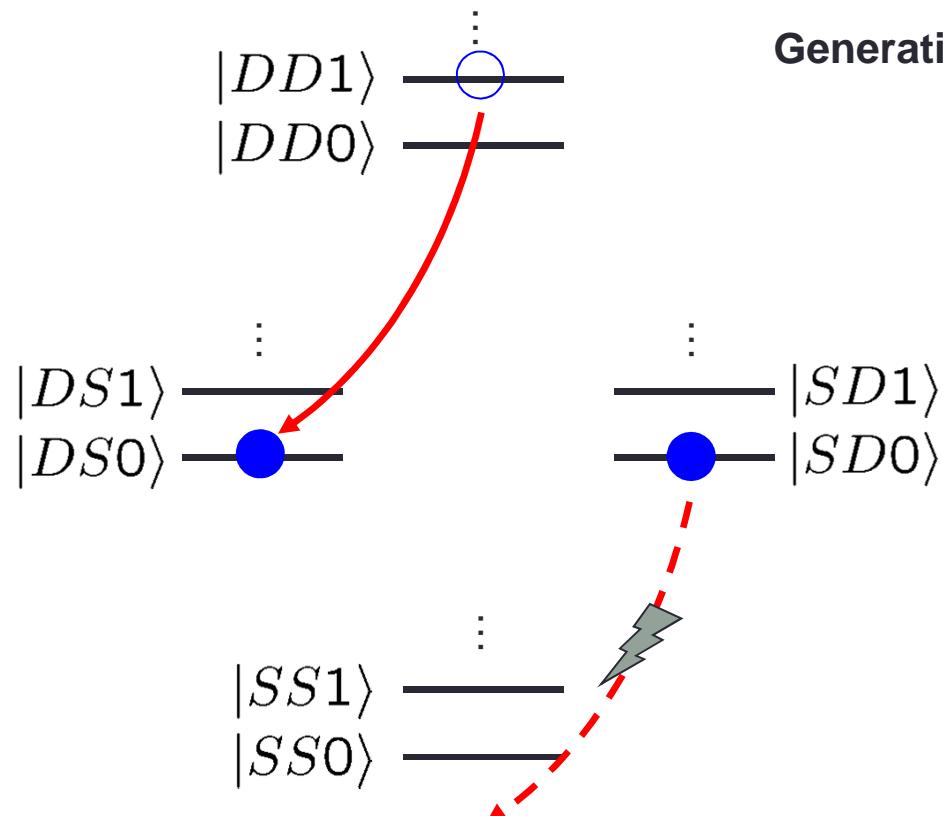
Ion 1:  $\pi/2$ , blue sideband

Ion 2:  $\pi$ , carrier

$$|SD0\rangle + |DD1\rangle$$



# Generation of Bell state



**Generation of Bell states:**  $|SD\rangle + |DS\rangle$

Pulse sequence:

Ion 1:  $\pi/2$ , blue sideband

Ion 2:  $\pi$ , carrier

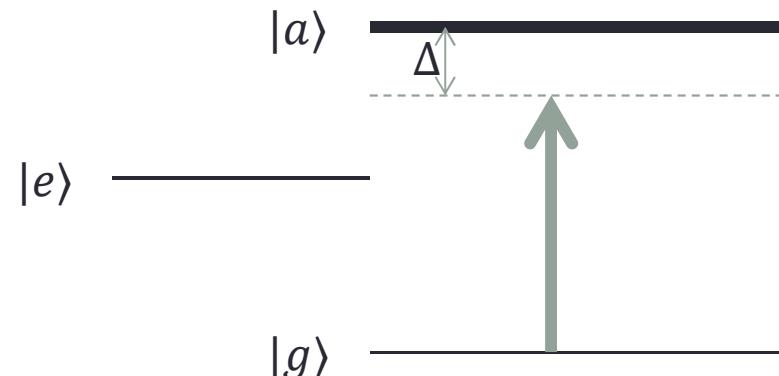
Ion 2:  $\pi$ , blue sideband

$$(|SD\rangle + |DS\rangle)|0\rangle$$



# Quantum phase

## AC Stark shift



$$\frac{\Delta E}{\hbar} = \frac{\Omega^2}{2\Delta}$$

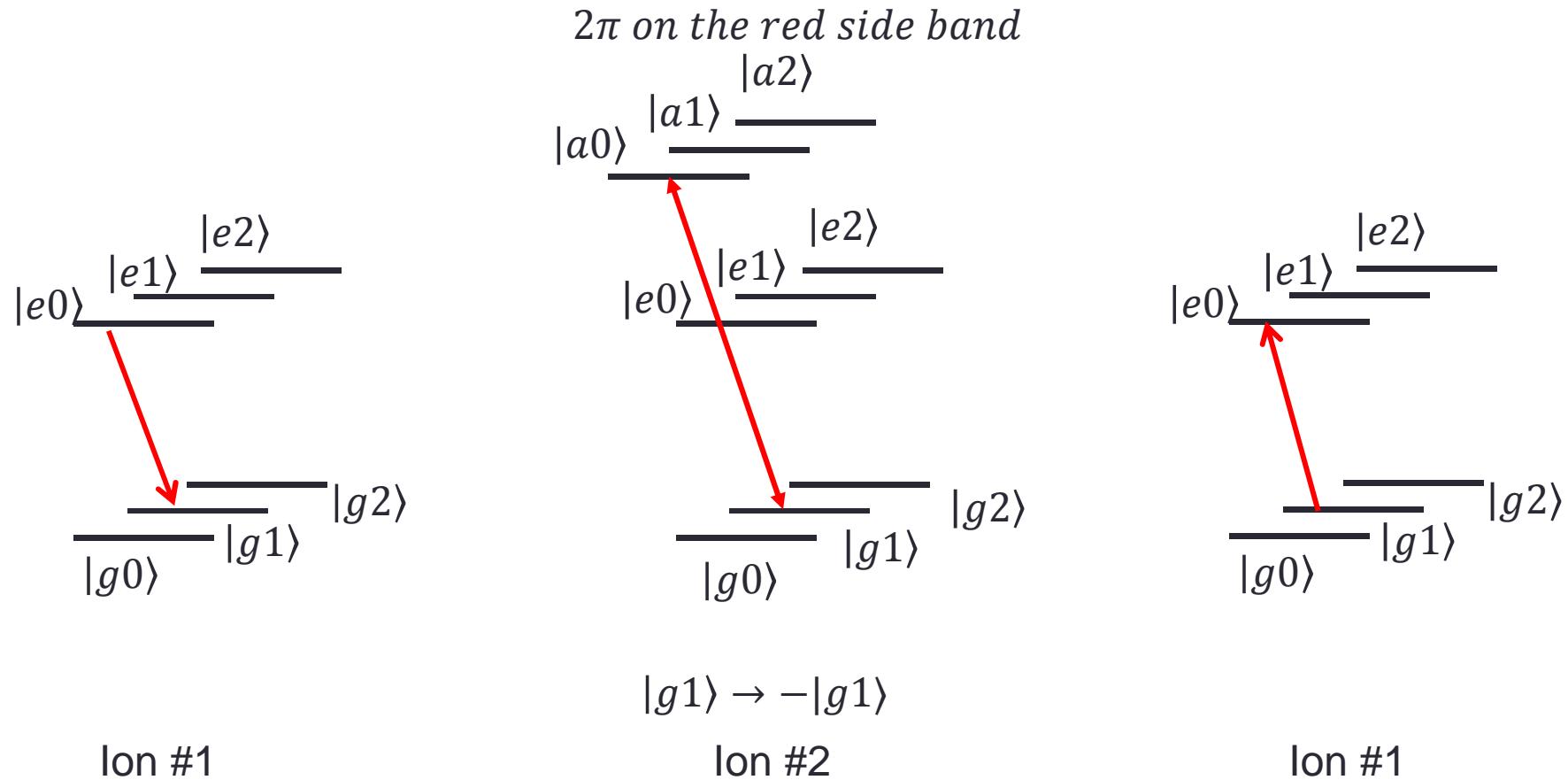
$$a|e\rangle + b|g\rangle \rightarrow |e\rangle + re^{-\frac{\Delta E}{\hbar}t}|g\rangle$$

Therefore we can use AC stark shift to generate relative phase



# Universal gate – C-NOT

Q-state manipulation – 2 qubit (original CZ gate)



$$|g1\rangle \rightarrow -|g1\rangle$$

Ion #1

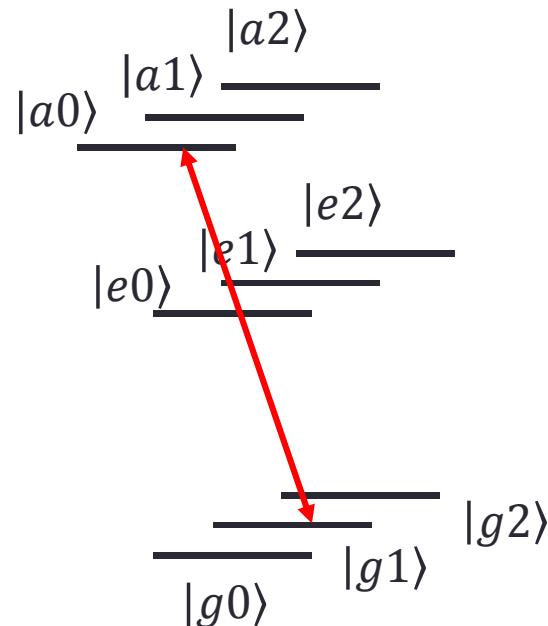
Ion #2

Ion #1

# Universal gate – C-NOT

Q-state manipulation – 2 qubit (original CZ gate)

*2π on the red side band*



Phase gate

$$|g1\rangle \rightarrow -|g1\rangle$$

$$|g0\rangle \rightarrow |g0\rangle$$

$$|e1\rangle \rightarrow |e1\rangle$$

$$|e0\rangle \rightarrow |e0\rangle$$

Ion #2



# Universal gate – C-NOT

Q-state manipulation – 2 qubit (original CZ gate)

Phase gate

$$|ee0\rangle \rightarrow -|ee0\rangle$$

$$|eg0\rangle \rightarrow |eg0\rangle$$

$$|ge0\rangle \rightarrow |ge0\rangle$$

$$|gg0\rangle \rightarrow |gg0\rangle$$

$$R\left(\frac{\pi}{2}, \phi_i\right)$$

$$|ee0\rangle \rightarrow |eg0\rangle$$

$$|eg0\rangle \rightarrow |ee0\rangle$$

$$|ge0\rangle \rightarrow |ge0\rangle$$

$$|gg0\rangle \rightarrow |gg0\rangle$$

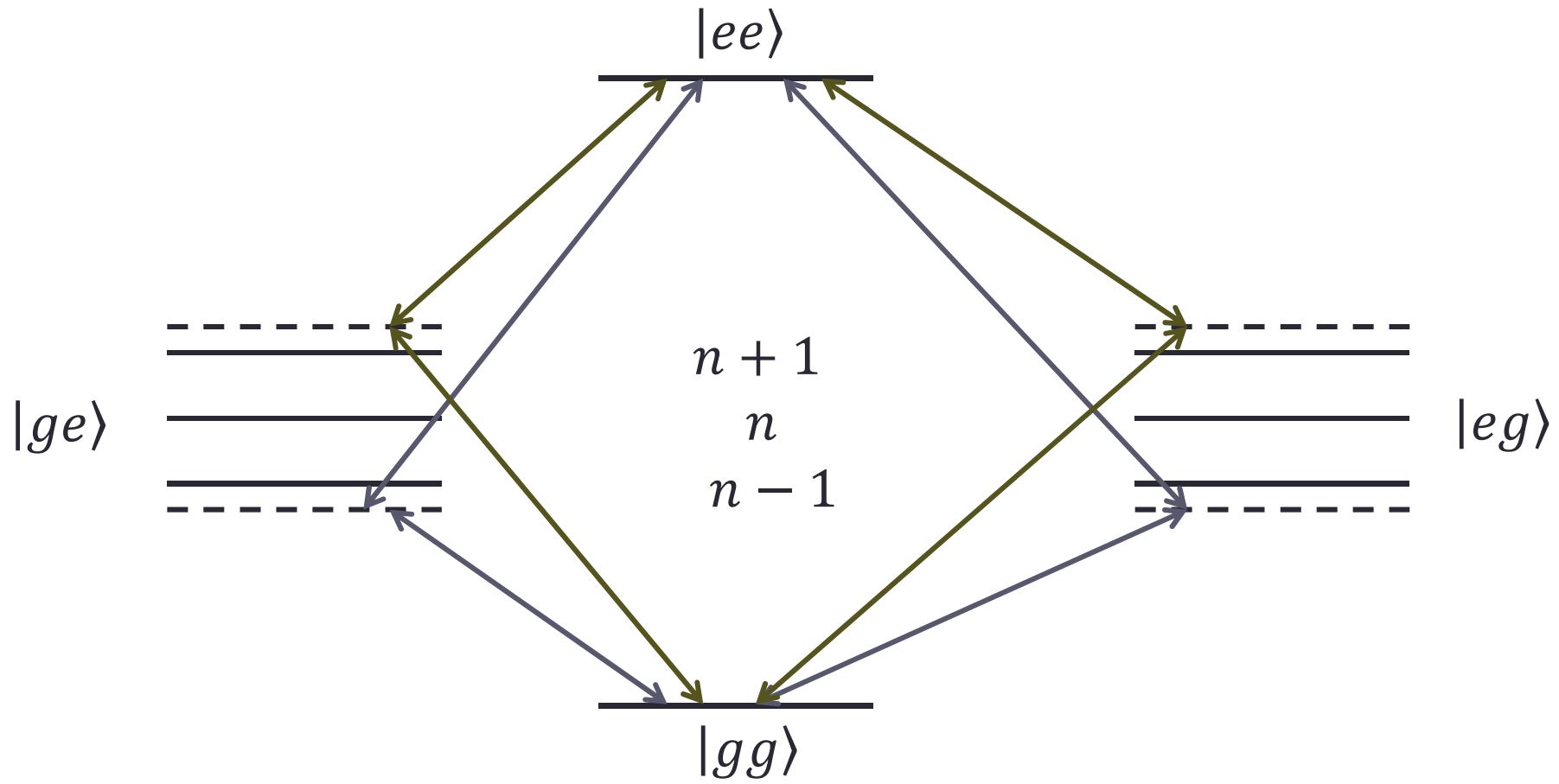


$a.$	$ g\rangle_m  g\rangle_n  0\rangle -$	$\rightarrow  g\rangle_m  g\rangle_n  0\rangle$
$b.$	$ g\rangle_m  e_0\rangle_n  0\rangle -$	$\text{C-NOT gate} \rightarrow  g\rangle_m  e_0\rangle_n  0\rangle$
$c.$	$ e_0\rangle_m  g\rangle_n  0\rangle -$	$\text{V}^{1/2}_n(\Pi/2) \text{PGate} \text{V}^{1/2}_n(-\Pi/2) \rightarrow  e_0\rangle_m  e_0\rangle_n  0\rangle$
$d.$	$ e_0\rangle_m  e_0\rangle_n  0\rangle -$	$\rightarrow  e_0\rangle_m  g\rangle_n  0\rangle$



# Universal gate

Q-state manipulation – 2 qubit (A. Sørensen & K. Mølmer)



# Universal gate

Q-state manipulation – 2 qubit (A. Sørensen & K. Mølmer)

$$|ee\rangle \rightarrow (|ee\rangle + i|gg\rangle)/\sqrt{2}$$

$$|gg\rangle \rightarrow (|gg\rangle + i|ee\rangle)/\sqrt{2}$$

$$|eg\rangle \rightarrow (|eg\rangle + i|ge\rangle)/\sqrt{2}$$

$$|ge\rangle \rightarrow (|ge\rangle + i|eg\rangle)/\sqrt{2}$$

Consider new basis as  $|\pm\rangle_i = (|e\rangle_i \pm |g\rangle_i)/\sqrt{2}$



# Universal gate

Q-state manipulation – 2 qubit (A. Sørensen & K. Mølmer)

$$|++\rangle \rightarrow |++\rangle$$

$$|--\rangle \rightarrow |--\rangle$$

$e^{\frac{i\pi}{4}}$  global phase ignored

$$|+-\rangle \rightarrow i|+-\rangle$$

$$|-+\rangle \rightarrow i|-+\rangle$$

It is universal along with single qubit rotation



# Universal gate

## Geometric phase gates – single qubit

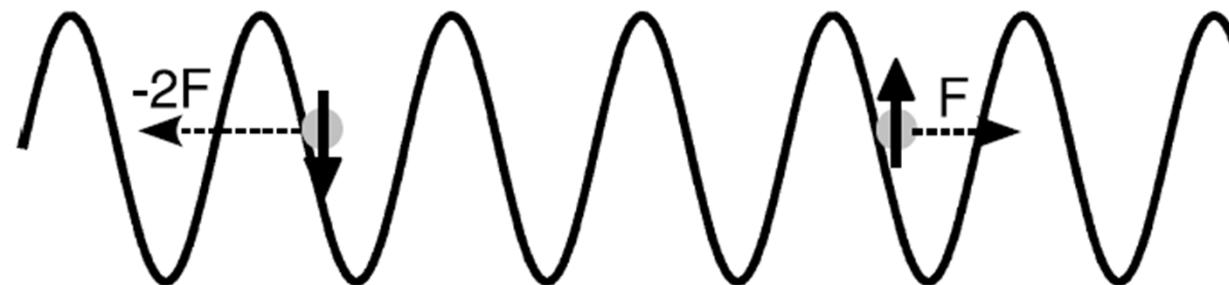
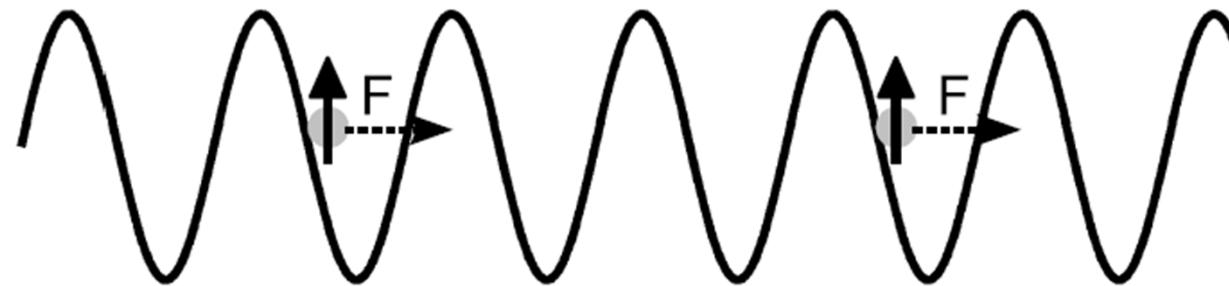
For single qubit these gates are usually produced by adiabatic rotation in Block sphere giving rise to Berry's phase.

For Universal operation, one needs both Abelian and non-Abelian phases



# Universal gate

Geometric phase gates – two qubit



# Universal gate

## Challenges in QC

Suppose we want to drive the sideband at a rate  $f\omega_s$  (similar to trap frequency) the coupling strength required is

$$\Omega = \frac{\Omega_+}{\eta} = \frac{f\omega_s}{\eta}$$

At this rate the AC Stark shift of the qubit will be

$$\frac{\Delta E}{\hbar} = \frac{\Omega^2}{2\Delta} = \frac{1}{2\omega_s} \frac{f\omega_s \Omega_+}{\eta^2} = \frac{f}{2\eta^2} \Omega_+$$

This means the phase evolution due to AC Stark shift becomes comparable to  $\Omega_+$  already for  $f = \eta^2$

Therefore (1) AC Stark shift (2) off resonant coupling  $\sim \frac{f^2}{\eta^2}$  are problematic



# Universal gate

## Challenges in QC

1. Gate fidelity
2. Gate time
3. Scalability

### References for day 4:

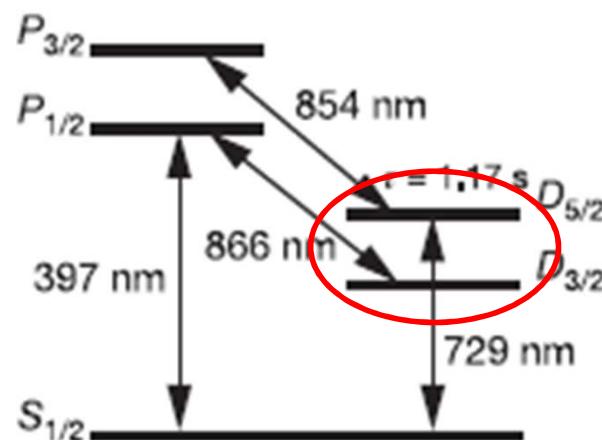
(look also for references within the review)

1. D. Leibfried et al. Rev. Mod. Phys. 75, 281 (2003)
2. Molmer and Sorensen Phys. Rev. Lett. 82, 1835 (1999)
3. D. Kielpinski et al. Nature 417, 709 (2002)



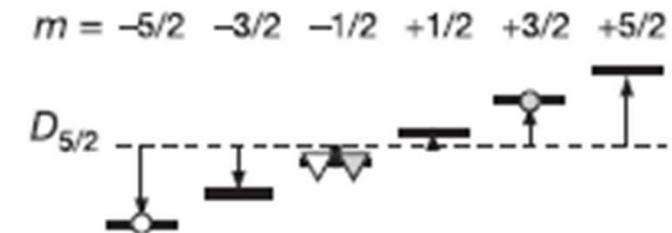
# Quantum metrology

## Entanglement in metrology: an example

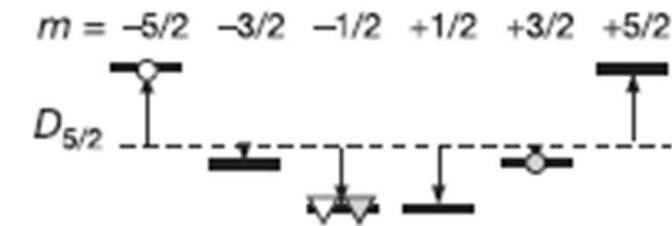


Ca atomic levels

### Application of magnetic field



### Calcium atomic level when trapped



C. F. Roos et al. Nature 443 316 (2006)

# Quantum metrology

## Entanglement in metrology: an example

b  $m = -5/2 \quad -3/2 \quad -1/2 \quad +1/2 \quad +3/2 \quad +5/2$



c  $m = -5/2 \quad -3/2 \quad -1/2 \quad +1/2 \quad +3/2 \quad +5/2$



$$\hbar\Delta\nu = \frac{1}{4} \frac{dE_z}{dz} \Theta(D, j) \frac{j(j+1) - 3m_j^2}{j(2j-1)} (3\cos^2\beta - 1)$$

Quadrupole shift measurement in  $\text{Ca}^+$

C. F. Roos et al. Nature 443 316 (2006)



# Quantum metrology

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|u_1\rangle|u_2\rangle + |v_1\rangle|v_2\rangle)$$

initial state

$$|\psi(\tau)\rangle = \frac{1}{\sqrt{2}}(|u_1\rangle|u_2\rangle + e^{i\lambda_\phi\tau}|v_1\rangle|v_2\rangle)$$

evolution of the state with time

$$\lambda_\phi = \frac{[(E_{u1}+E_{u2})-(E_{v1}+E_{v2})]}{\hbar}$$

measuring the phase provides  
information about the energy difference

De-coherence free sub-space (DFS) chosen as

$$|\psi(\tau)\rangle = \frac{1}{\sqrt{2}}\left(\left|-\frac{5}{2}\right\rangle\left|+\frac{3}{2}\right\rangle + \left|-\frac{1}{2}\right\rangle\left|-\frac{1}{2}\right\rangle\right)$$

Not affected by magnetic field  
fluctuations



# Quantum metrology

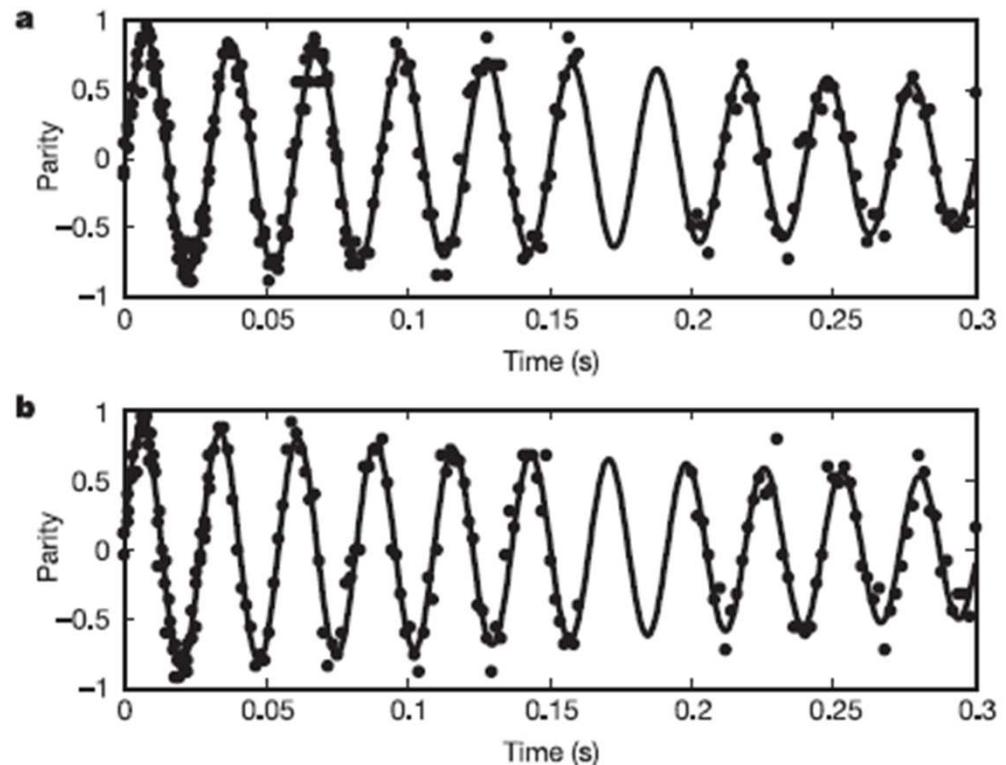
## Entanglement in metrology: an example

Parity measurement:

$$P = \hat{P}_{++} + \hat{P}_{--} - \hat{P}_{+-} - \hat{P}_{-+}$$

$\hat{P}_{\pm\pm}$  implies projection on to  
 $|\pm\rangle_1 \times |\pm\rangle_2$       where

$$|\pm\rangle_{k=1,2} = \frac{1}{\sqrt{2}} (|u_k\rangle \pm |v_k\rangle)$$



Quadrupole shift measurement in  $\text{Ca}^+$   
C. F. Roos et al. Nature 443 316 (2006)

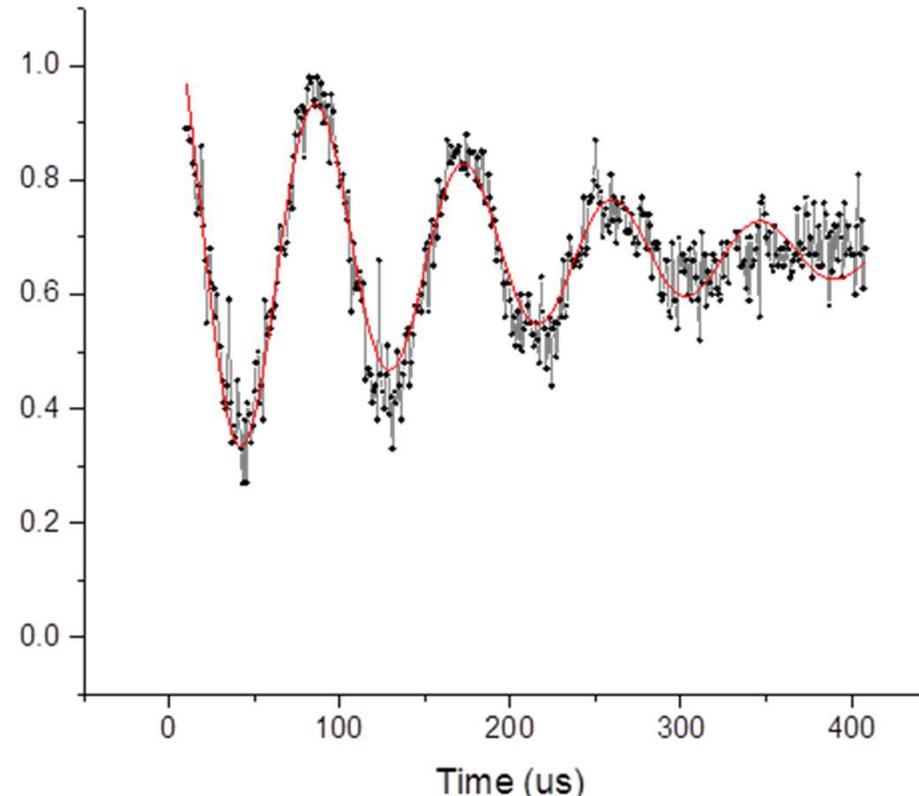


# Quantum metrology

Illness of magnetic field fluctuation: an example

Parity measurement:

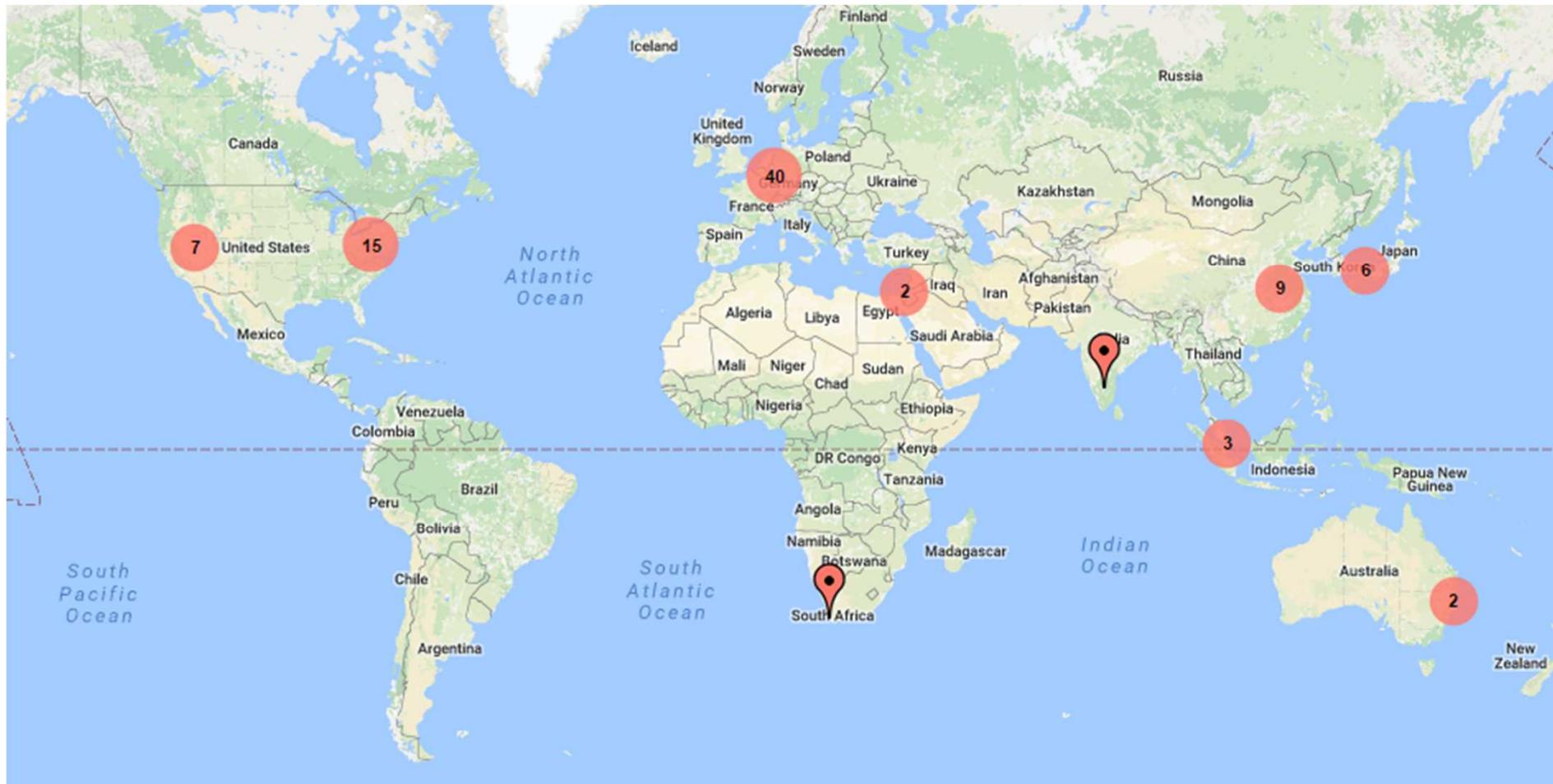
$$P = P_e - P_g$$



Ramsey measurement on Ba<sup>+</sup>



# The world of traps



# References:

## Books:

1. Quantum Computation and Quantum Information, Book by Isaac Chuang and Michael Nielsen, Cambridge press
2. Ion traps, book by P. K. Ghosh, Oxford press
3. Principles of ion traps, G. Werth, Springer

## Reviews:

1. Leibfried et al. Rev. Mod. Phys. 75, 281 (2003)
2. A. D. Ludlow et al. Rev. Mod. Phys. 87, 637 (2015)

And all the references there in :-)

