Paul trap – eq. of motion

Approximation as per the real linear trap operation:

- Axial confinement is weaker than radial $a_x < q_x$
- Works close to the origin of 1st stability region $|a_x|,\ q_x^2 \ll 1$
- Assuming $C_{\pm 4} \approx 0$

One obtains: $$\beta_x \approx \sqrt{a_x + \frac{q_x^2}{2}}$$

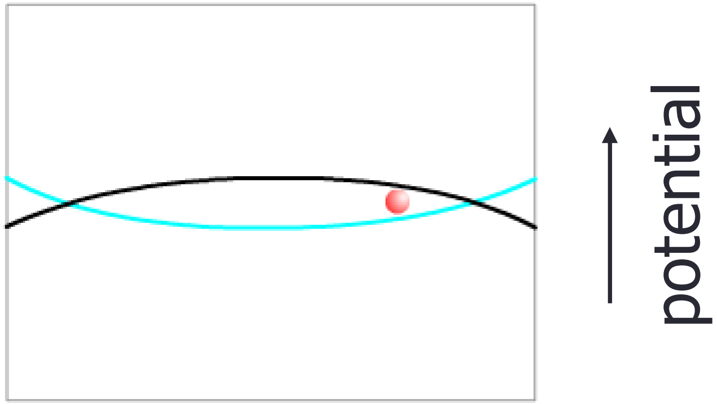
And : $$x(t) \approx 2AC_0 \cos\left(\frac{\beta_x \Omega}{2} t\right)\left[1 - \frac{q_x}{2}\cos(\Omega t)\right]$$

$$= 2AC_0 \cos\left(\frac{\beta_x \Omega}{2} t\right) - \frac{2AC_0 q_x}{2} \cos \frac{\beta_x \Omega}{2} t \cos \Omega t$$

Micro motion

Secular motion

Paul trap – eq. of motion



$$x(t) = 2AC_0 \cos\left(\frac{\beta_x \Omega}{2} t\right) - \frac{2AC_0 q_x}{2} \cos\frac{\beta_x \Omega}{2} t \cos \Omega t$$

Secular motion

Micro motion

Pseudo-potential approximation

The mean displacement of the ion is negligible within time $\frac{1}{\Omega}$

The total displacement is composed of secular and micro-motion parts

$x = x_s + x_\mu$ reminder $\boxed{x(t) \approx 2AC_0 \cos\left(\frac{\beta_x \Omega}{2} t\right)\left[1 - \frac{q_x}{2}\cos(\Omega t)\right]}$

Secular displacement is large but frequency is slow as compared to micro-motion

$x_s \gg x_\mu$ $\dot{x}_s \ll \dot{x}_\mu$

The time-dependent motion in x is reminder $\boxed{\ddot{x} + (a_x - 2q_x cos2\zeta)x = 0}$

$\ddot{x_\mu} = -(a_x - 2q_x cos2\zeta)\, x_s$ (1) $\boxed{a_x \ll q_x}$

Integrating over time

$x_\mu = -\frac{q_x}{2} cos2\zeta\, x_s$ $\boxed{x_s \text{ is constant in one period}}$

Pseudo-potential approximation

Therefore the amplitude of motion is:

$$x = x_s - \frac{q_x}{2} cos2\zeta \, x_s$$

Substituting in $\ddot{x} + (a_x - 2q_x cos2\zeta)x = 0$:

$$\ddot{x} = -(a_x - 2q_x cos2\zeta)\left(1 - \frac{q_x}{2} cos2\zeta\right) x_s$$

$$= -a_x x_s - q_x^2 \cos^2 2\zeta \, x_s + 2q_x \cos 2\zeta \, x_s + \frac{q_x a_x}{2} \cos 2\zeta \, x_s$$

Averaging over one cycle of RF:

$$< \ddot{x_s} >= -\left(a_x + \frac{q_x^2}{2}\right) x_s$$

$$\left\langle \frac{d^2 x_s}{dt^2} \right\rangle = -\left(a_x + \frac{q_x^2}{2}\right) \frac{\Omega^2}{4} x_s$$

Reminder: $\boxed{\zeta = \frac{\Omega t}{2}}$

Pseudo-potential approximation

From pseudo potential model one obtains:

$$\left\langle \frac{d^2 x_s}{dt^2} \right\rangle = -\left(a_x + \frac{q_x^2}{2}\right)\frac{\Omega^2}{4} x_s = -\frac{\beta_x^2 \Omega^2}{4} x_s = -\omega_x^2\, x_s$$

From solving the Mathieu equation one obtains :

$$x\,(t) = 2AC_0 \cos\left(\frac{\beta_x \Omega}{2} t\right) - \frac{2AC_0 q_x}{2} \cos \frac{\beta_x \Omega}{2} t \cos \Omega t$$

Therefore they match and we observe that the motion is a simple harmonic oscillator motion

Problem 5: Prove that the pseudo-potential trap depth is $\bar{D}_x = \frac{eV_0^2}{4mr_0^2\Omega^2}$ *considering* $a_x = 0$.

QM treatment

The trap potential may be written as :

$$\hat{V}(t) = \frac{m}{2} W(t)\hat{x}^2$$ where $$W(t) = \frac{\omega_{rf}^2}{4}\left[a_x + 2q_x \cos(\omega_{rf} t)\right]$$

With these definition the Hamiltonian looks :

$$\hat{H}^m = \frac{\hat{p}^2}{2m} + \frac{m}{2} W(t)\hat{x}^2$$

Reminder: $u(\zeta) = Ae^{\{i\beta_x\zeta\}}\Sigma_{\{n=-\infty\}}^{\infty} C_{2n}e^{\{i2n\zeta\}} + Be^{\{-i\beta_x\zeta\}}\Sigma_{\{n=-\infty\}}^{\infty} C_{2n}e^{\{-i2n\zeta\}}$

QM treatment

$$\widehat{H}^m = \frac{\hat{p}^2}{2m} + \frac{m}{2} W(t) \hat{x}^2$$

The equation of motion of the operators in Heisenberg picture are:

$$\dot{\hat{x}} = \frac{1}{i\hbar}\left[\hat{x}, \widehat{H}^m\right] = \frac{\hat{p}}{m} \qquad \dot{\hat{p}} = \frac{1}{i\hbar}\left[\hat{p}, \widehat{H}^m\right] = -mW(t)\hat{x}$$

By combining we obtain:

$$\ddot{\hat{x}} + W(t)\hat{x} = 0$$

This is equivalent to Mathieu equation (not surprising!!) provided $\hat{x}$_is replace by u(t) function. So to solve this Hamiltonian, we use the special solution of Mathieu equation subject to boundary conditions

Reminder: $u(\zeta) = Ae^{\{i\beta_x\zeta\}}\Sigma_{\{n=-\infty\}}^{\infty} C_{2n} e^{\{i2n\zeta\}} + Be^{\{-i\beta_x\zeta\}}\Sigma_{\{n=-\infty\}}^{\infty} C_{2n} e^{\{-i2n\zeta\}}$

QM treatment

Reminder: $u(\zeta) = Ae^{\{i\beta_x\zeta\}}\Sigma_{\{n=-\infty\}}^{\infty} C_{2n}e^{\{i2n\zeta\}} + Be^{\{-i\beta_x\zeta\}}\Sigma_{\{n=-\infty\}}^{\infty} C_{2n}e^{\{-i2n\zeta\}}$

$u(0) = 1, \qquad \dot{u}(0) = iv$

These boundary condition implies $A = 1, B = 0$

$$u(t) = e^{\frac{i\beta_x\omega_{rf}t}{2}} \Sigma_{n=-\infty}^{\infty} C_{2n}e^{in\omega_{rf}t} = e^{\frac{i\beta_x\omega_{rf}t}{2}} \Phi(t)$$

Periodic with period $T = \frac{2\pi}{\omega_{rf}}$

Therefore the coefficients takes the form:

$$\Sigma_{n=-\infty}^{\infty} C_{2n} = 1$$

$u(0) = 1$

$$v = \omega_{rf}\, \Sigma_{n=-\infty}^{\infty} C_{2n}\left(\frac{\beta_x}{2} + n\right)$$

$\dot{u}(0) = iv$

This solution and its complex conjugate are linearly independent and hence they obey Worskian identity

QM treatment

This solution and its complex conjugate are linearly independent and hence they obey Wronskian identity

$$u^*(t)\dot{u}(t) - u(t)\dot{u}^*(t) = u^*(0)\dot{u}(0) - u(0)\dot{u}^*(0) = 2i\nu$$

Similar argument holds for $\hat{x}(t)$ and $u(t)$ as both obey the same differential equations, so a complex linear combination as

$$\hat{C}(t) = \sqrt{\frac{m}{2\hbar\nu}}\, i\{u(t)\dot{\hat{x}}(t) - \dot{u}(t)\hat{x}(t)\}$$

Is also proportional to their Wronskian identity and also constant in time

QM treatment

$$\hat{C}(t) = \hat{C}(0) = \sqrt{\frac{1}{2m\hbar\nu}}\{m\nu\hat{x}(0) + i\hat{p}(0)\}$$

This is familiar annihilation operator of static HO of mass m and frequency ν

$\hat{C}(t) = \hat{C}(0) = \hat{a}$ Implies $[\hat{C}, \hat{C}^T] = [\hat{a}, \hat{a}^T] = 1$

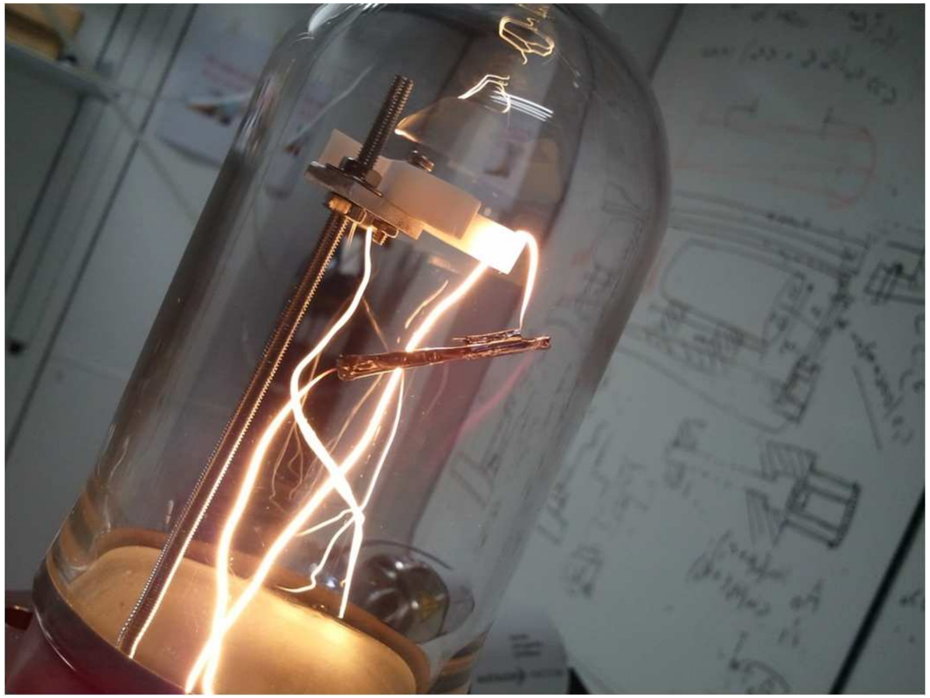
This oscillator which is time independent is known as the reference oscillator

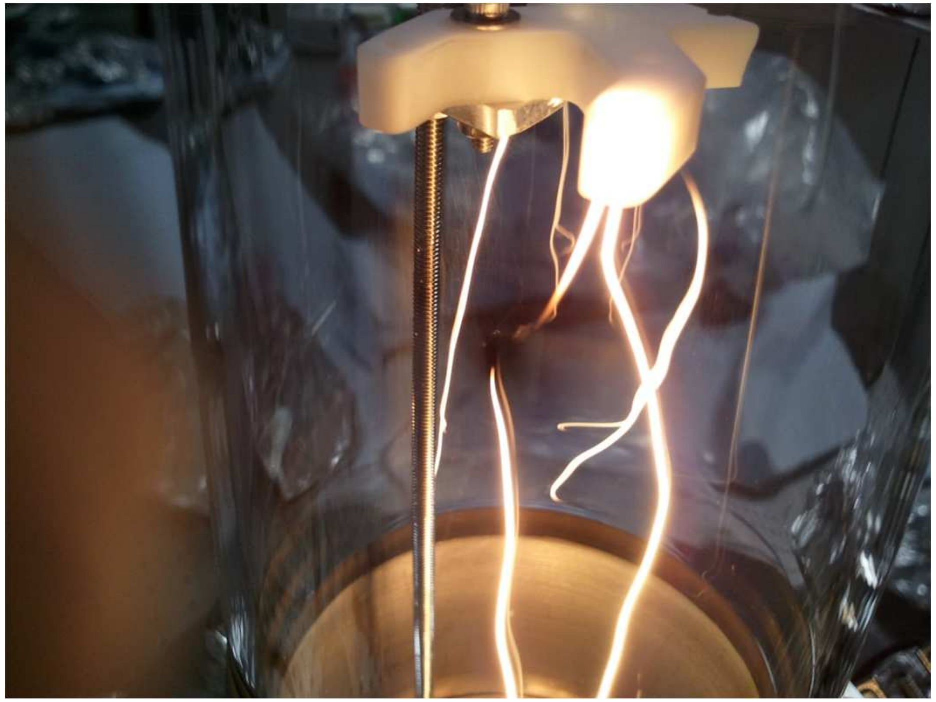
$$\hat{x}(t) = \sqrt{\frac{\hbar}{2m\nu}}\{\hat{a}u^*(t) + \hat{a}^T u(t)\}$$

$$\hat{p}(t) = \sqrt{\frac{\hbar m}{2\nu}}\{\hat{a}\dot{u}^*(t) + \hat{a}^T \dot{u}(t)\}$$

Ion trapping – the steps

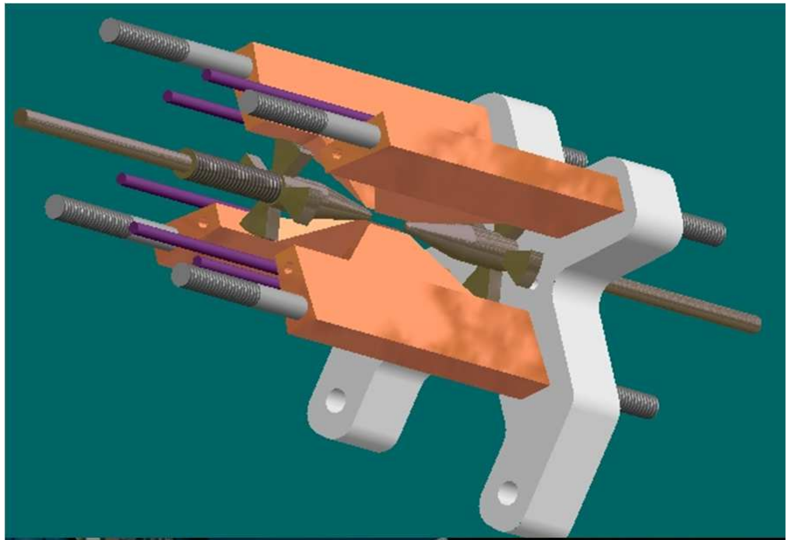
Atomic oven – resistive heating

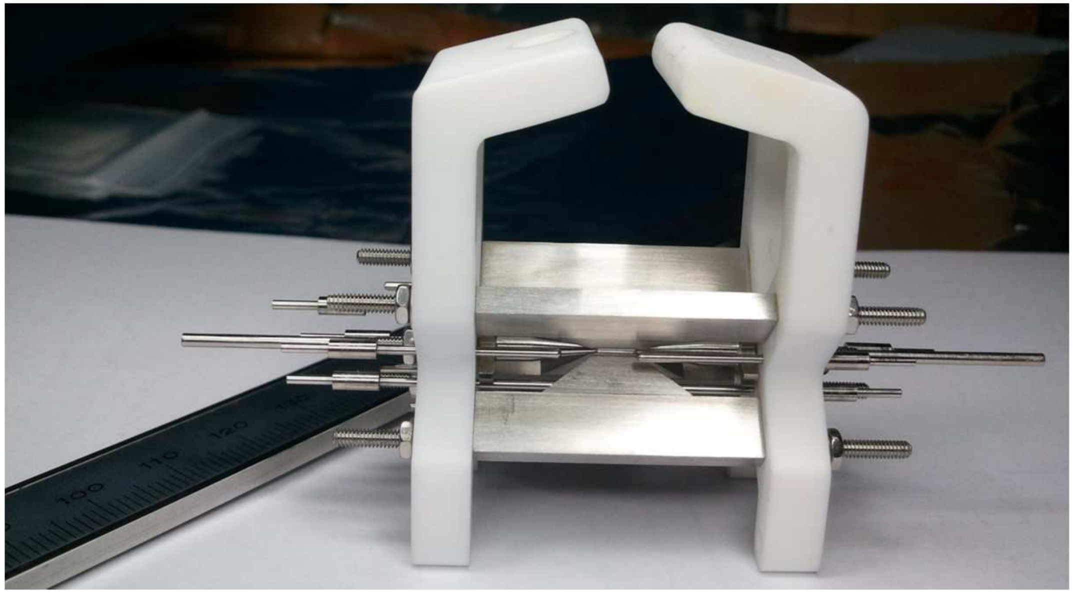




Ion trapping – the steps

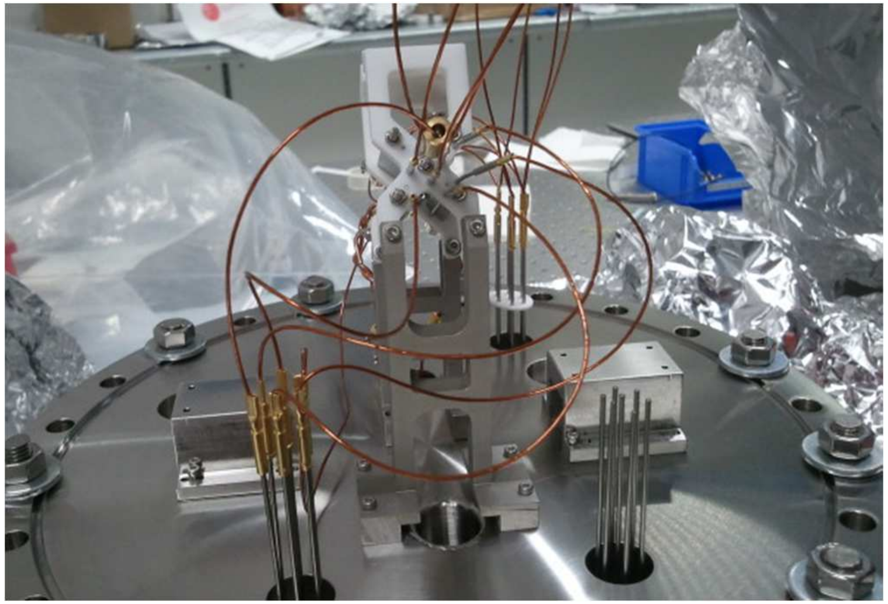
Trap assembly – UHV protocols

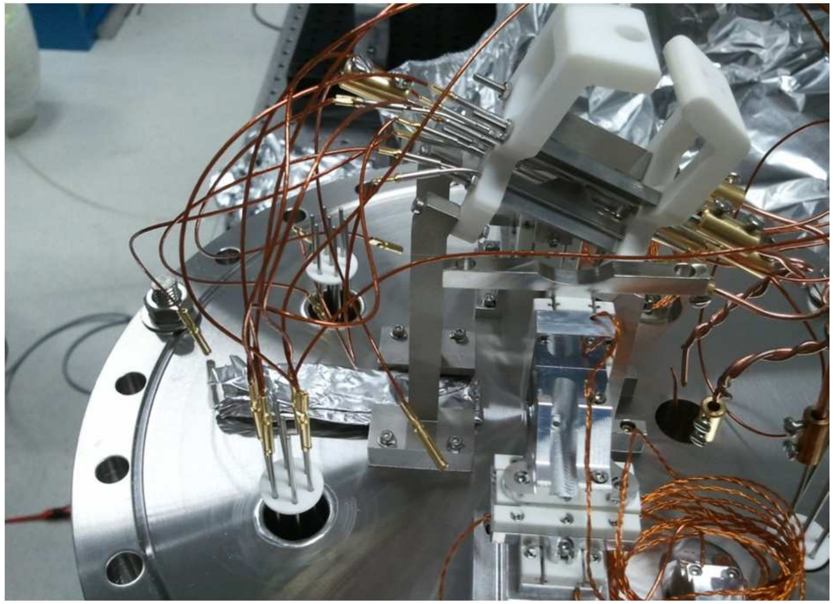




Ion trapping – the steps

Trap assembly – UHV protocols



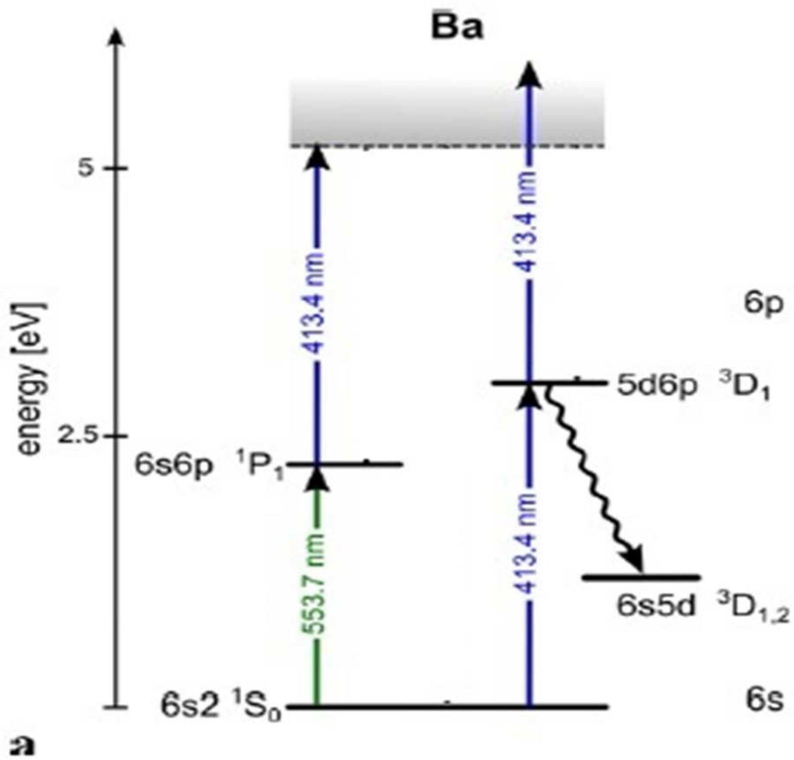


Ion trapping – the steps

Ion creation – in-situ

1. Electron impact
2. Surface ionization
3. Resonant laser ionization
4. etc.

Example for Ba^+

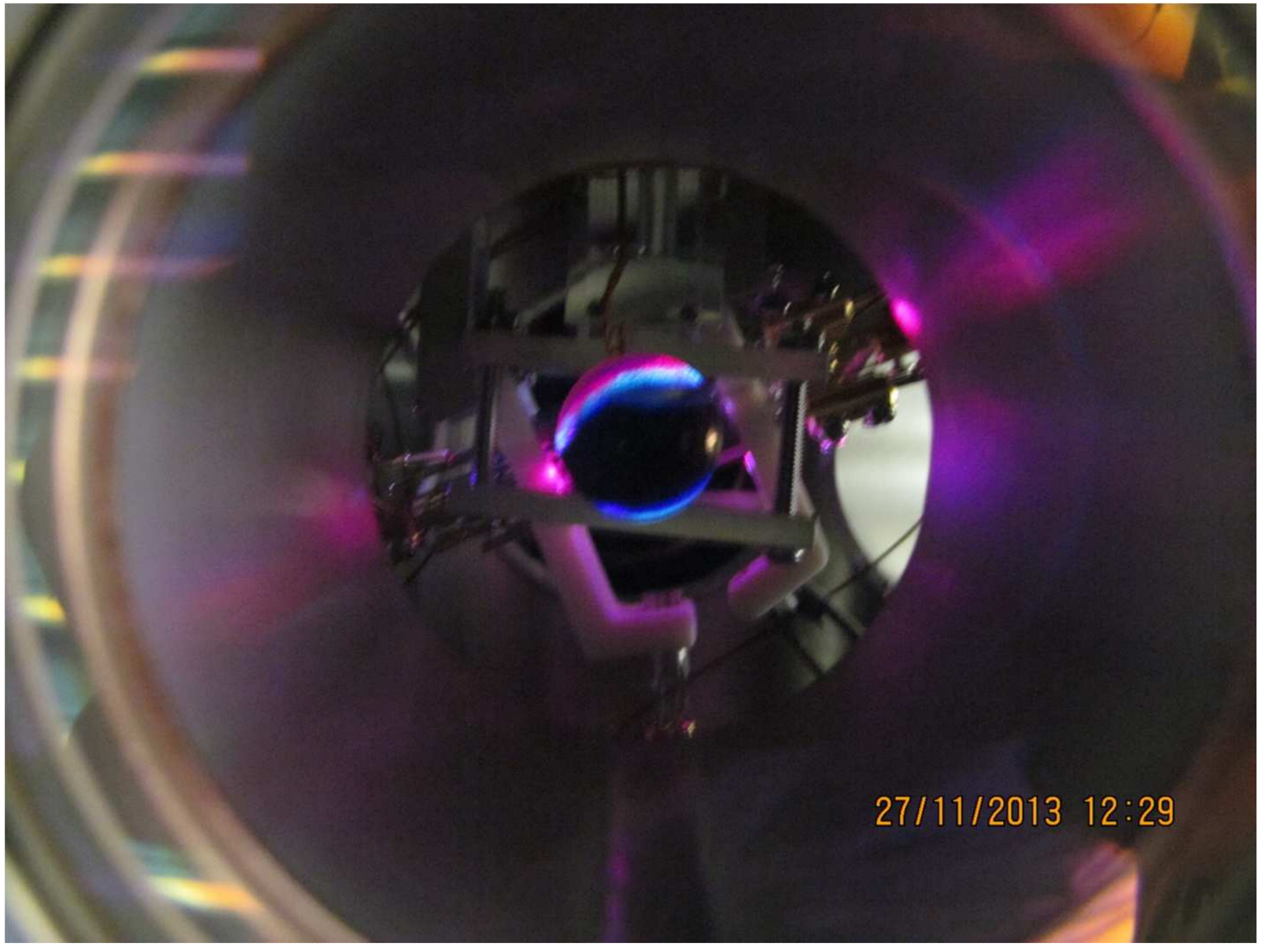


Ion trapping – the steps

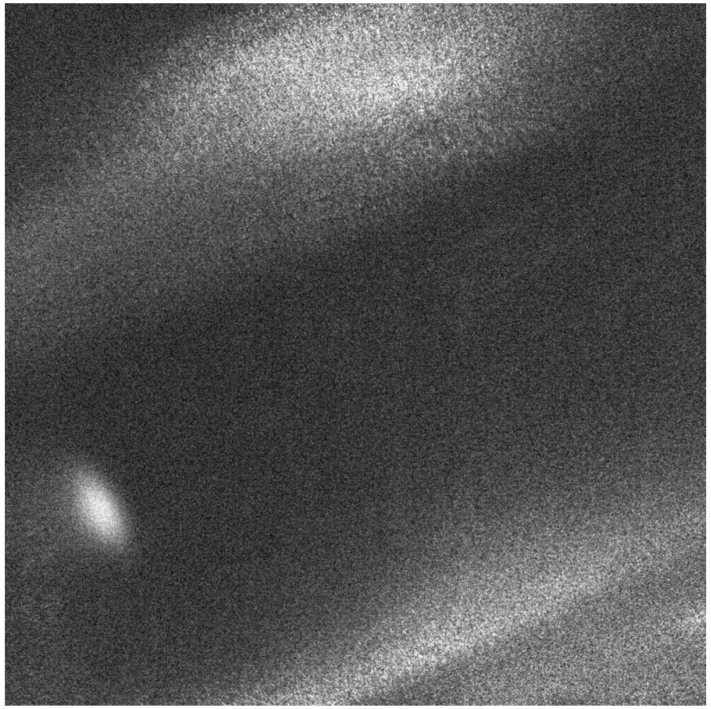
Ion trap drive

Ion trapping – the steps

Ion imaging

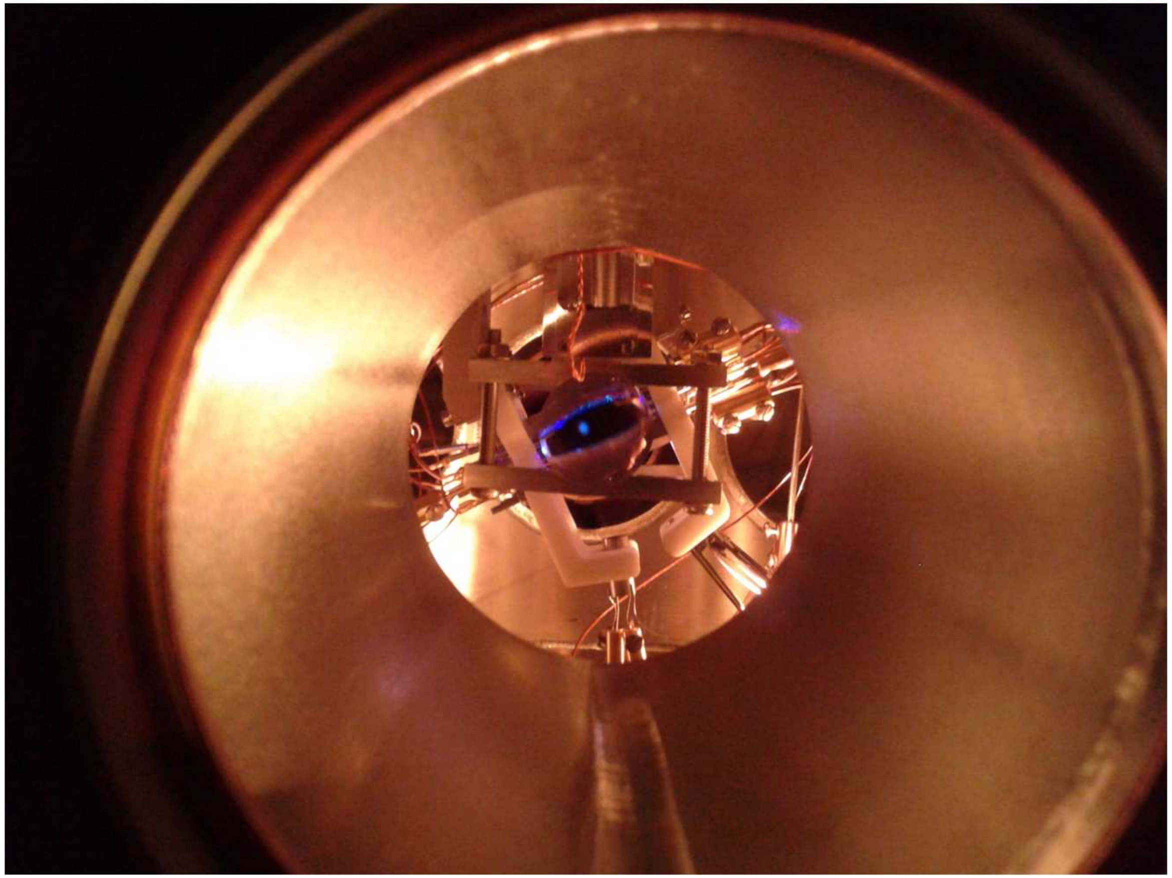


Ion trapping – the steps



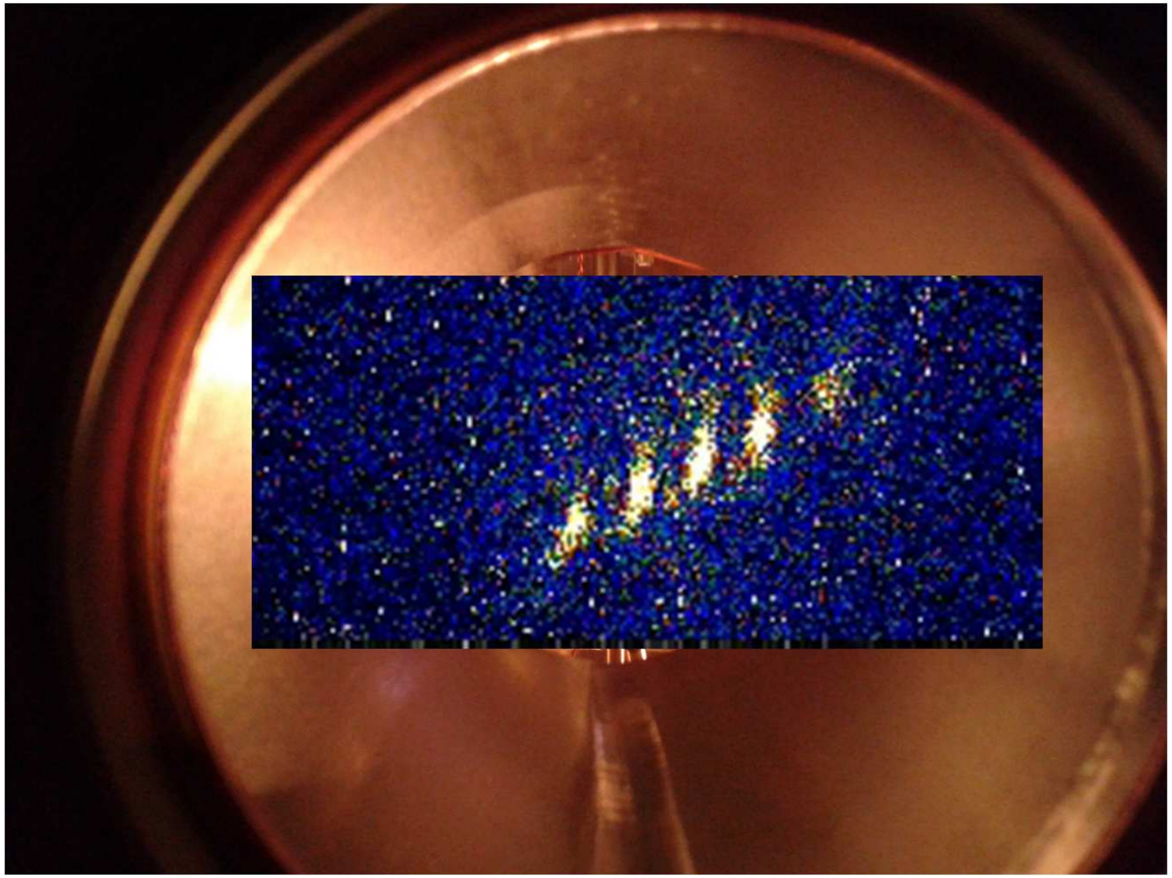
Is there ion?

Ion trapping – the steps



Smart phone image – not Apple!!

Ion trapping – the next steps



A EM CCD image

MAKING OF A QUBIT

Content

1. Light matter interaction
2. Cooling of ions
3. Single qubit operations
4. Multi-qubit operations

Light matter interaction

Time dependent SE

$$i\hbar \frac{\partial|\psi(\vec{r},t)\rangle}{\partial t} = H(t)|\psi(t)\rangle$$

$$i\hbar \frac{\partial|\psi\rangle}{\partial t} = [H_0 + H'(t)]|\psi\rangle$$

atom Light-atom

Stationary states of atom

$$H_0|\phi_k\rangle = E_k|\phi_k\rangle$$

Any state in the atomic basis

$$|\psi\rangle = \Sigma_k c_k|\phi_k\rangle$$

Plugging back to TDSE

$$i\hbar \frac{\partial}{\partial t}\Sigma_k c_k|\phi_k\rangle = [H_0 + H'(t)]\Sigma_k c_k|\phi_k\rangle$$

Multiplying both sides by $\langle\phi_j|$ on both sides

$$i\hbar \frac{\partial}{\partial t} c_j(t) = \Sigma_k H'_{jk} c_k(t) e^{i\omega_{jk}t}$$

$$\boxed{\begin{aligned} H'_{jk} &= \langle\phi_j|H'(t)|\phi_k\rangle \\ \omega_{jk} &= (\omega_j - \omega_k) \end{aligned}}$$

$$i\hbar \frac{\partial}{\partial t} c_j(t) = \Sigma_k H'_{jk} c_k(t) e^{i\omega_{jk} t}$$

This equation is exact but not possible to solve without approximating

We are interested in laser light interacting with an atom. Therefore assuming the laser to be of single frequency and addressing only two states of the atom. Therefore truncate the summation to only two states:

<u>Two-level system interacting with light:</u>

$$i\hbar \frac{dc_g(t)}{dt} = c_e(t) H'_{ge}(t) e^{-i\omega_a t}$$
$$i\hbar \frac{dc_e(t)}{dt} = c_g(t) H'_{eg}(t) e^{i\omega_a t}$$

$j = g; k = e$ ground and excited state
$\omega_a = \omega_e - \omega_g$ atomic resonance frequency

Now we need to calculate the exact form of $H'_{ge}(t)$ for light matter interaction (2-level approximation)

$$H = \frac{P^2}{2m} + V(r)$$

KE

Coulomb energy

Reminder (EM-II):

$$\vec{A}(\vec{r},t) = \left(A_0\hat{\epsilon_z}\, e^{i(ky-\omega t)} + A_0^*\hat{\epsilon_z}\, e^{-i(ky-\omega t)}\right)$$

$$\frac{E}{2} = -\frac{\partial A}{\partial t} = i\omega A_0$$

$$\frac{B}{2} = \nabla \times A = ikA_0$$

$$H = \frac{(P-eA)^2}{2m} + V(r) - \frac{e}{m}\vec{S}\cdot\vec{B}$$

$$= \frac{P^2}{2m} + V(r) + \frac{e^2A^2}{2m} - \frac{e}{2m}\left(\vec{P}\cdot\vec{A} + \vec{A}\cdot\vec{P}\right) - \frac{e}{m}\vec{S}\cdot\vec{B}$$

B field spin interaction

Energy of EM field

E field - charge interaction

$$H = \frac{(P - eA)^2}{2m} + V(r) - \frac{e}{m}\vec{S}\cdot\vec{B}$$

$$= \frac{P^2}{2m} + V(r) + \cancel{\frac{e^2A^2}{2m}} - \frac{e}{2m}\left(\vec{P}\cdot\vec{A} + \vec{A}\cdot\vec{P}\right) - \cancel{\frac{e}{m}\vec{S}\cdot\vec{B}}$$

Neglect A^2 as compared to A

Neglect $\vec{S}.\vec{B}$ as compared to $\vec{P}.\vec{A}$

$$= H_0 - \frac{e}{m}\vec{P}\cdot\vec{A}$$

$$= H_0 - \frac{e}{m}P_z\left[A_0 e^{iky}e^{-i\omega t} - A_0^* e^{-iky}e^{i\omega t}\right]$$

Dipole approximation only 1st term is kept

Expanding the exponential factor

$$e^{\pm iky} = e^{\pm\frac{i2\pi y}{\lambda}} = 1 \pm iky - \frac{k^2y^2}{2}\cdots$$

$$= H_0 - \frac{eE}{m\omega}P_z \sin\omega t$$

By proper choice of gauge it can be shown to be equivalent to

$$= H_0 - e\vec{E}.\vec{r}$$

$$= H_0 + H_I$$

American Journal of Physics 50, 128 (1982)

Two most important interactions are electric dipole and electric quadrupole

$$H_D = e\vec{E}.\vec{r}$$

$$H_Q = eQ_{ij}\frac{\partial E_i}{\partial x_j} = \frac{1}{2}e\left(x_i x_j - \frac{1}{3}\delta_{ij}x^2\right) = \frac{1}{2}e\,k\,z(\vec{E}.\vec{z})\left[ie^{i(ky-\omega t)} + c.c.\right]$$

In matrix representation in the basis of $|e\rangle$ and $|g\rangle$

$$H_{D/Q} = \hbar\Omega_0^{D/Q}(\,|g\rangle\langle e|+|e\rangle\langle g|) \times [e^{i(kx-\omega t+\phi)} + e^{-i(kx-\omega t+\phi)}].$$

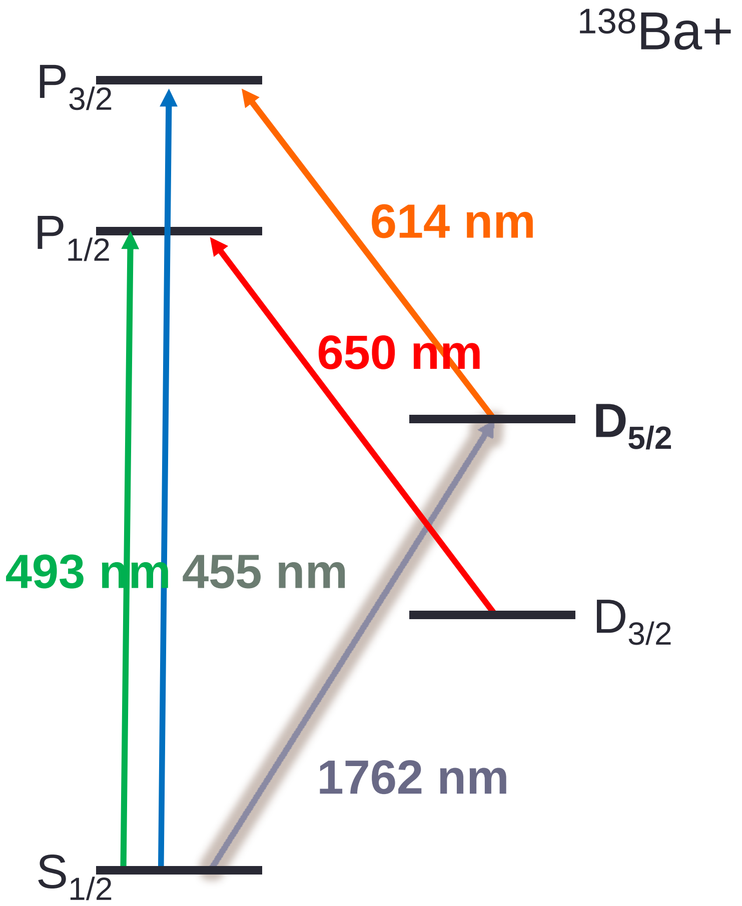
Where,

$$\frac{\hbar}{2}\Omega_0^D = e\langle g|\vec{E}\cdot\vec{r}|e\rangle$$ Dipole transition

$$\frac{\hbar}{2}\Omega_0^Q = \frac{ek}{2}\left\langle g\,\middle||\vec{r}|(\vec{E}\cdot\vec{r})\middle|\,e\right\rangle$$ Quadrupole transition

$$\frac{\hbar}{2}\Omega_0^{RT} = -\hbar\frac{|\Omega_{g3}\Omega_{e3}|}{\Delta_R}e^{i\Delta\phi}$$ Two photon Raman transition

138Ba+ Ion Energy Level @ $D_{5/2}$

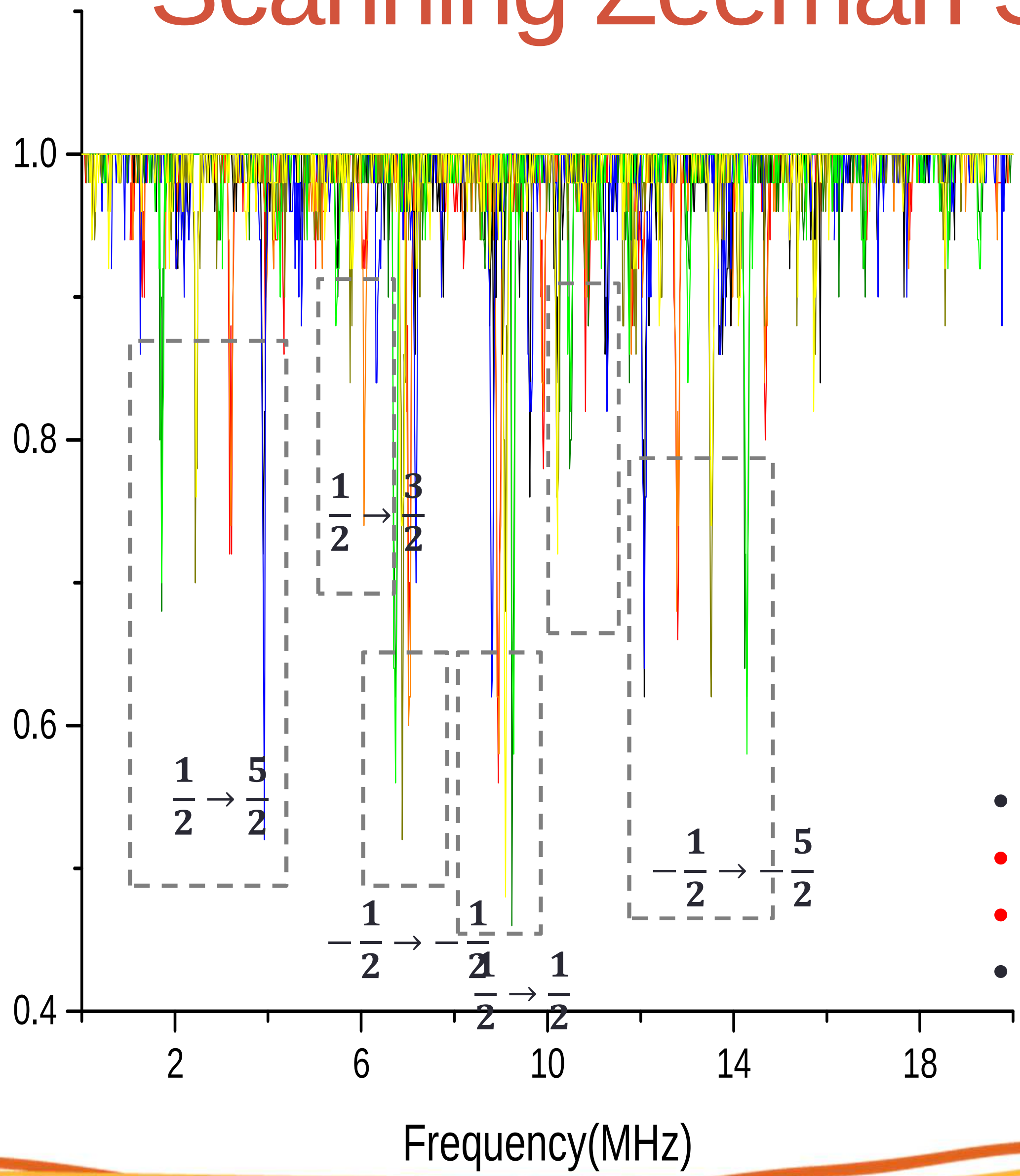


- **$S_{1/2}$ - $D_{5/2}$ transition**

  1762.1745 nm (170.12643 THz)

  τ : 34.5 sec.

Scanning Zeeman States



Blue/Black	2 A
Red/Orange	2.5 A
Yellow/Brown	3 A
Green	3.5 A

- Scan 20 MHz range.
- B field – Laser beam : 45°
- B field – beam polarization : 45°
- X axis is the frequency detuning.

Total Hamiltonian: trapped 2-level ion

$$H = H_{atom} + H_{trap} + H_I$$
$$= H_0 + H_I$$

Atomic Hamiltonian:

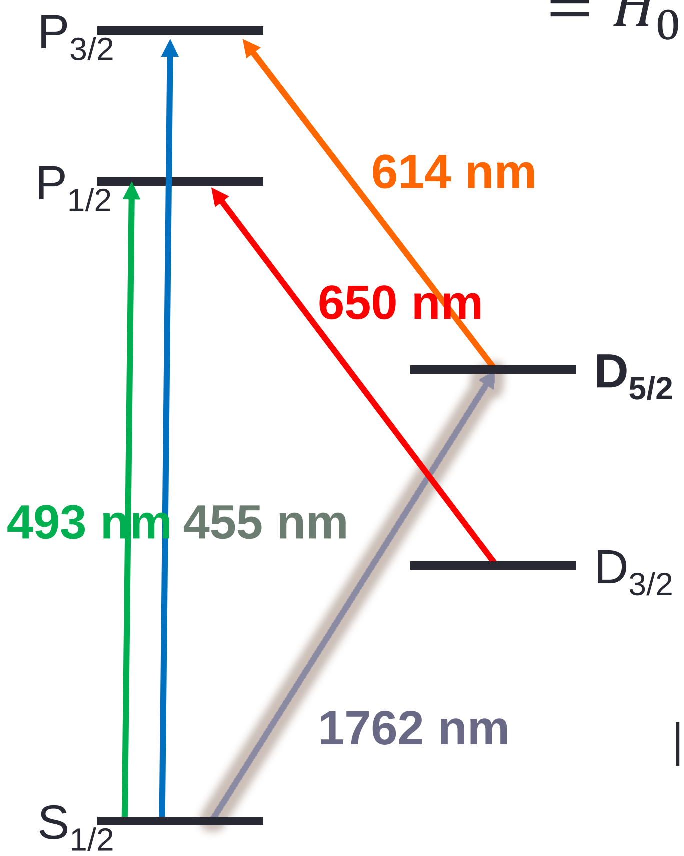
$$H_{atom} = \frac{\hbar}{2}\omega_a(\,|e\rangle\langle e| - |g\rangle\langle g|\,) = \frac{\hbar}{2}\omega_a\sigma_z$$

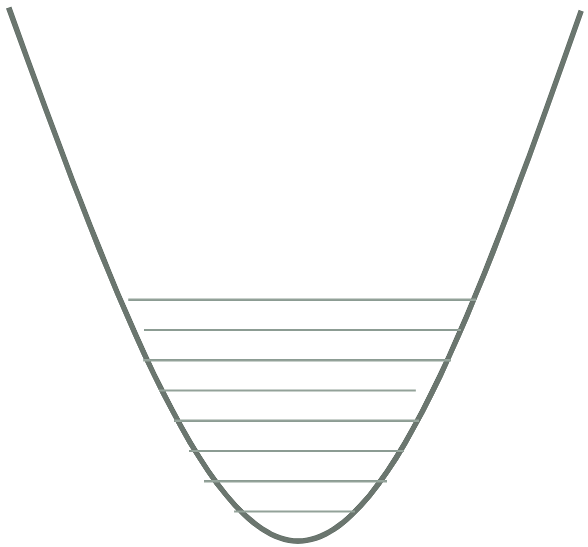
$$H_{trap} = \frac{\hat{p}^2}{2m} + \frac{m}{2}\left(\frac{\Omega_{rf}^2}{4}(\,a_x + 2q_x\cos(\Omega_{\mathrm{rf}}t))\right)\hat{x}^2$$

The solution of the unperturbed Hamiltonian is completely known, therefore

Total Hamiltonian: trapped 2-level ion

$$H = H_{atom} + H_{trap} + H_I$$
$$= H_0 + H_I$$

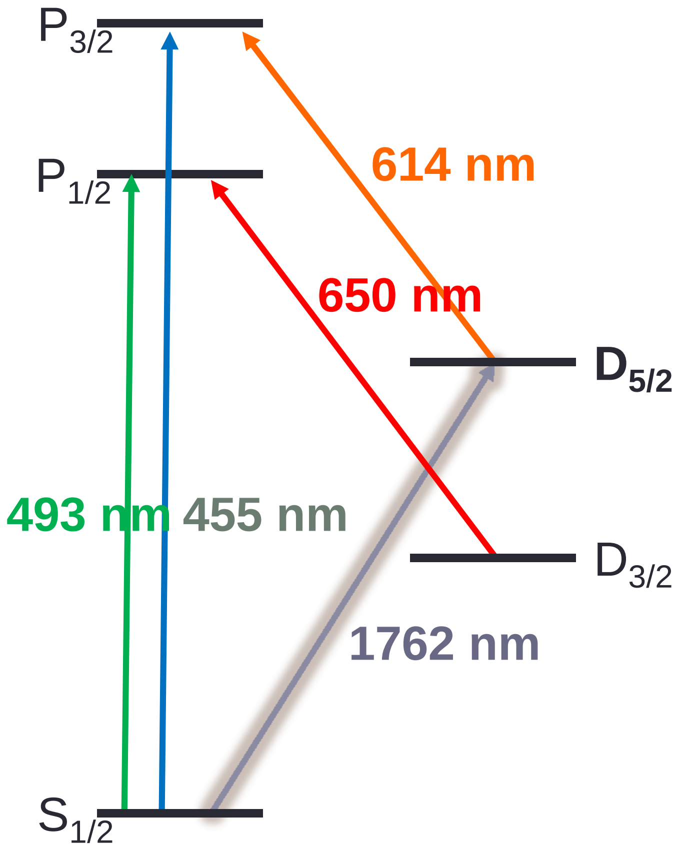


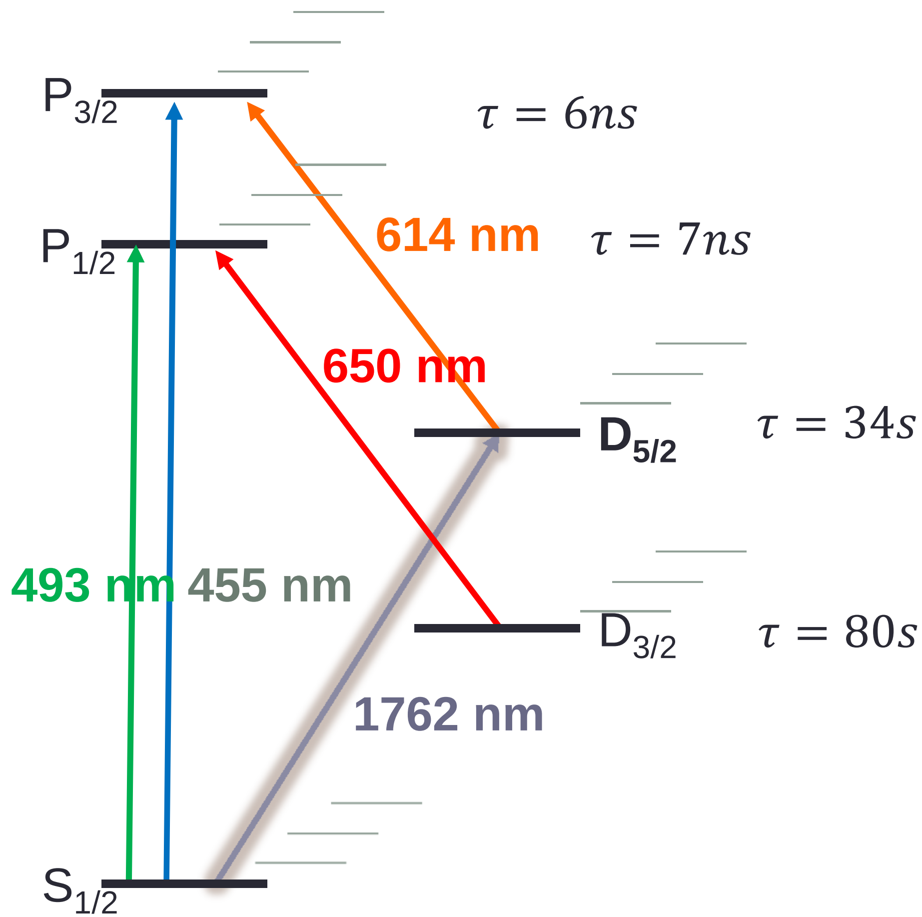


$$|\psi\rangle_{sys} = |\psi\rangle_{at} \times |\psi\rangle_{trap}$$

Total Hamiltonian: trapped 2-level ion

The concept





H_I in interaction picture is:

Reminder $U_0 = e^{-\frac{i}{\hbar}\widehat{H_0}\,t}$

$$H_{inter} = {U_0}^{\dagger} H_I U_0$$

$$= \frac{\hbar}{2}\Omega e^{\frac{i}{\hbar}H_{atom}t}(\sigma_+ + \sigma_-)e^{-\frac{i}{\hbar}H_{atom}t} \times e^{\frac{i}{\hbar}H_{trap}t}\left[\, e^{i(k\hat{x}-\omega t+\phi)} + e^{-i(k\hat{x}-\omega t+\phi)}\right]e^{-\frac{i}{\hbar}H_{trap}t}$$

Baker–Campbell–Hausdorff formula
Reminder: $e^{X}Ye^{-X} = Y + [X,Y] + \frac{1}{2!}\left[X,[X,Y]\right] + \cdots$

$$= \frac{\hbar}{2}\Omega\left(\sigma_+ e^{i\omega_a t} + \sigma_- e^{-i\omega_a t}\right) \times e^{\frac{i}{\hbar}H_{trap}t}\left[\, e^{i(k\hat{x}-\omega t+\phi)} + e^{-i(k\hat{x}-\omega t+\phi)}\right]e^{-\frac{i}{\hbar}H_{trap}t}$$

- Rotating wave approximation for $(\omega_a \pm \omega)$
- The transformation of H_{trap} to interaction picture is same as converting to

  Heisenberg picture meaning $\hat{x} \rightarrow \hat{x}(t)$

$$\hat{x}(t) = \sqrt{\frac{\hbar}{2m\nu_{sec}}}\left(\hat{a}u^*(t) + \hat{a}^\dagger u(t)\right)$$

Therefore we obtain:

$$H_{inter} = \frac{\hbar}{2}\Omega\sigma_+ e^{i\left(\phi+\eta\left[\hat{a}u^*(t)+\hat{a}^\dagger u(t)\right]-\delta t\right)} + h.c.$$

Lamb-Dicke parameter;

$$\eta = k\sqrt{\frac{\hbar}{2m\nu_{sec}}}$$

$$H_{inter} = \frac{\hbar}{2}\Omega\sigma_+ e^{i(\phi+\eta[\hat{a}u^*(t)+\hat{a}^\dagger u(t)]-\delta t)} + h.c.$$

Further simplification can be done by considering the parameter regime in which a linear trap works:

$$\boxed{\begin{array}{l}(|a_x|, q_x^2) \ll 1 \equiv \beta_x \omega_{rf} \sim \nu \\ C_0 \sim \left(1+\frac{q_x}{2}\right)^{-1}\end{array}}$$

$$H_{inter}(t) = \frac{\frac{\hbar}{2}\Omega}{1+\frac{q_x}{2}}\sigma_+ e^{i\eta(\hat{a}e^{-i\nu t}+\hat{a}^\dagger e^{i\nu t})} e^{i(\phi-\delta t)} + h.c.$$

$$= \frac{\hbar}{2}\Omega_0\sigma_+ e^{i\eta(\hat{a}e^{-i\nu t}+\hat{a}^\dagger e^{i\nu t})} e^{i(\phi-\delta t)} + h.c.$$

Special case: Lamb-Dicke regime

Spread of the wave packet~ 10nm

$$\eta = k\sqrt{\frac{\hbar}{2m\nu_{sec}}} = \frac{2\pi}{\lambda}\sqrt{\frac{\hbar}{2m\nu_{sec}}}$$

Wavelength of probe light~ 500 nm

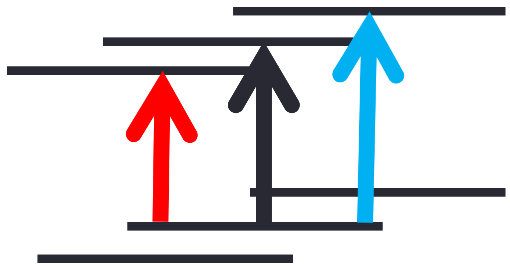
$$\boxed{\eta \ll 1}$$

$$H_{inter}(t) = \frac{\hbar}{2}\Omega_0\sigma_+\left(1 + i\eta\left(\hat{a}e^{-i\nu t} + \hat{a}^\dagger e^{i\nu t}\right)\right)e^{i(\phi-\delta t)} + h.c.$$

Three cases of importance:

Carrier ($\delta = 0$):

$$H_c = \frac{\hbar}{2}\Omega_0(\sigma_+ e^{i\phi} + \sigma_- e^{-i\phi})$$



Red Side Band (RSB) ($\delta = -\nu$):

$$H_c = \frac{\hbar}{2}\Omega_0\eta(\hat{a}\sigma_+ e^{i\phi} + \hat{a}^\dagger\sigma_- e^{-i\phi})$$

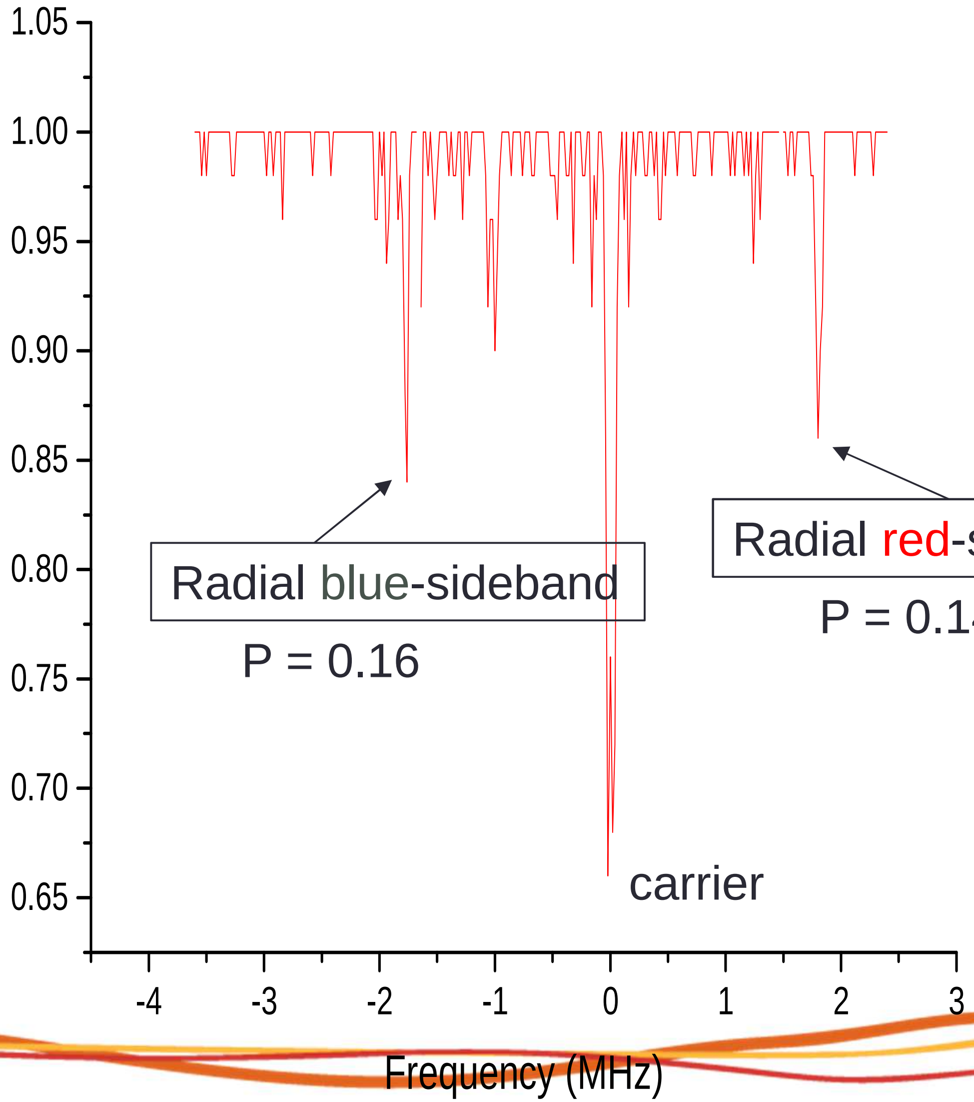
$$\Omega_{n,n-1} = \Omega_0\sqrt{n}\eta$$

Blue Side Band (BSB) ($\delta = +\nu$):

$$H_c = \frac{\hbar}{2}\Omega_0\eta(\hat{a}^\dagger\sigma_+ e^{i\phi} + \hat{a}\sigma_- e^{-i\phi})$$

$$\Omega_{n,n+1} = \Omega_0\sqrt{n+1}\eta$$

Carrier and Side-band



Power of rf = 4 W
(trap frequency ~ 1.8 MHz)

$$\frac{P\_red}{P\_blue} = \frac{n}{n+1} = \frac{0.14}{0.16}$$

n = 7

Higher order sidebands $\delta = l\nu$ with $|l| \geq 1$

These involve two-"phonons"

$$H_c = \frac{\hbar}{2}\Omega_0 \frac{\eta^2}{2}\left(\hat{a}^2\sigma_+ e^{i\phi} + \hat{a}^{\dagger 2}\sigma_- e^{-i\phi}\right)$$

For $l = -2$, since it depends on η^2 the coupling strength is very low

Resolved sideband

$\delta = l\nu + \delta'$ where $\delta' \ll \nu$

$$|\psi(t)\rangle = \Sigma_{n=0}^{\infty} c_{n,g}(t)|n,g\rangle + c_{n,e}(t)|n,e\rangle$$

$$i\hbar\partial_t|\psi(t)\rangle = \hat{H}_{int}|\psi(t)\rangle \qquad \textit{[time dependent SE]}$$

$$\dot{c}_{n,g} = -i^{1-|l|}e^{i(\delta' t-\phi)}\left(\frac{\Omega_{n+l,n}}{2}\right)c_{n+l,e}$$
$$\dot{c}_{n+l,e} = -i^{1+|l|}e^{-i(\delta' t-\phi)}\left(\frac{\Omega_{n+l,n}}{2}\right)c_{n,g}$$

Laplace transform to solve:

$$\begin{bmatrix} c_{(n+l,e)}(t) \\ c_{(n,g)}(t) \end{bmatrix} = T_n^l \begin{bmatrix} c_{n+l,e}(0) \\ c_{n,g}(0) \end{bmatrix}$$

Solutions to TDSE

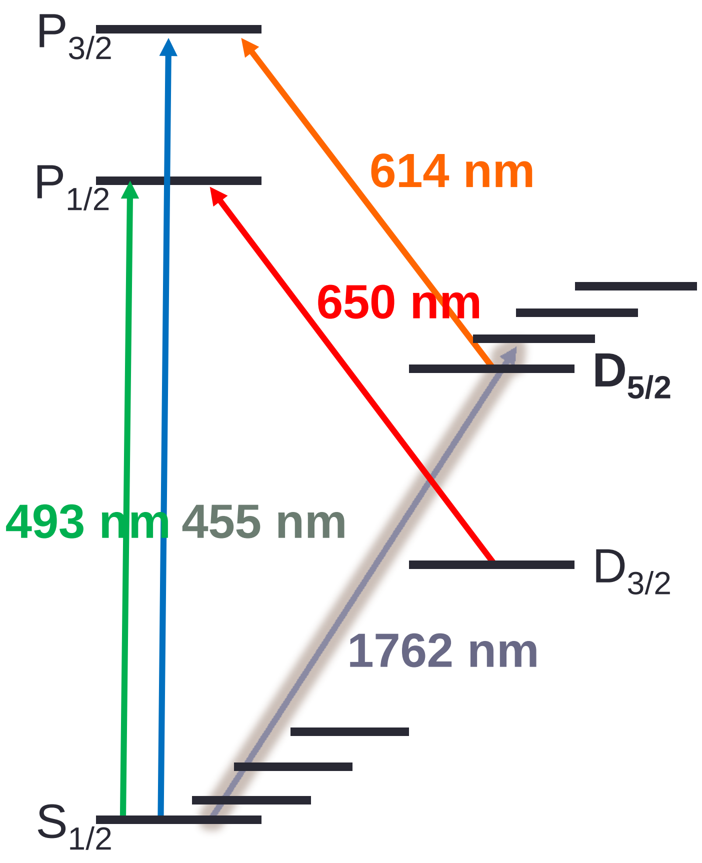
Resolved sideband

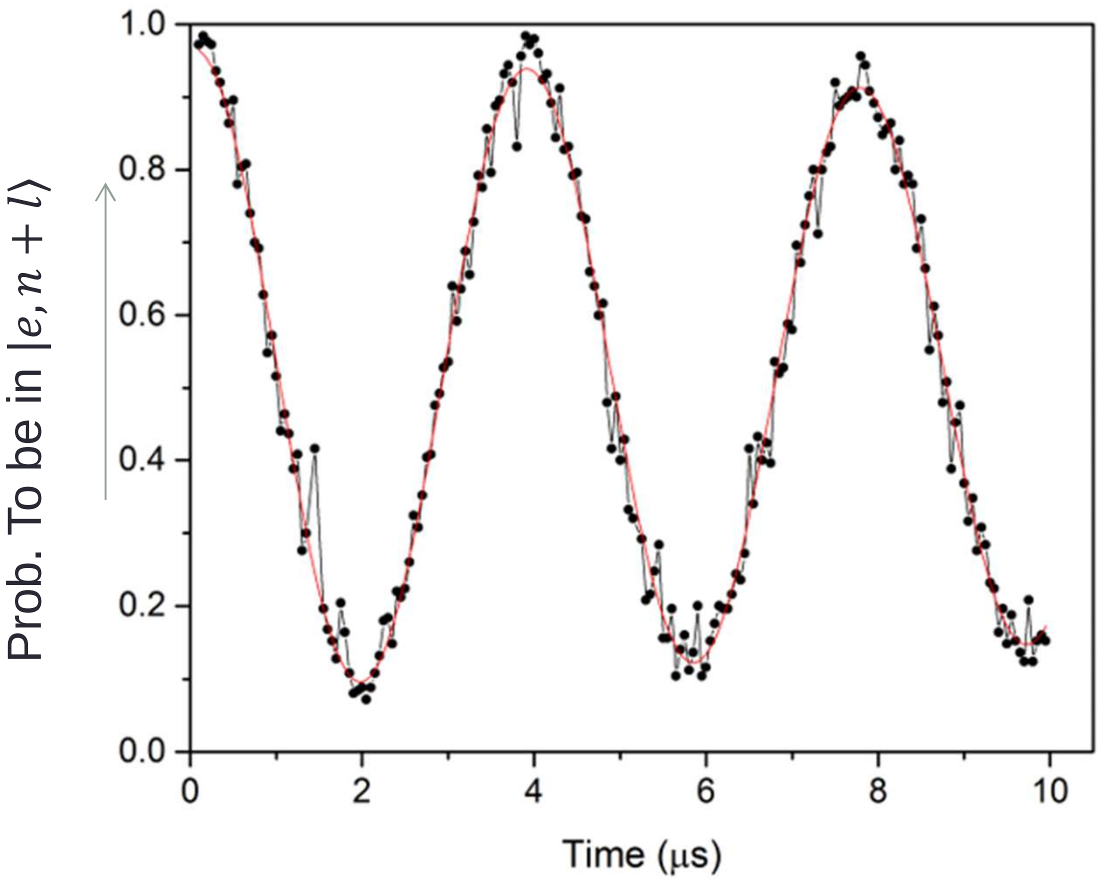
$$T_n^l = \begin{pmatrix} e^{-i\left(\frac{\delta'}{2}\right)t}\left[\cos\left(\frac{f_n' t}{2}\right) + \frac{i\delta'}{f_n'}\sin\left(\frac{f_n' t}{2}\right)\right] & -i\frac{\Omega_{n+l,n}}{f_n^l}e^{i\left(\phi + \frac{|l|\pi}{2} - \frac{\delta' t}{2}\right)}\sin\left(\frac{f_n' t}{2}\right) \\ -i\frac{\Omega_{n+l,n}}{f_n^l}e^{-i\left(\phi + \frac{|l|\pi}{2} - \frac{\delta' t}{2}\right)}\sin\left(\frac{f_n' t}{2}\right) & e^{i\left(\frac{\delta'}{2}\right)t}\left[\cos\left(\frac{f_n' t}{2}\right) - \frac{i\delta'}{f_n'}\sin\left(\frac{f_n' t}{2}\right)\right] \end{pmatrix}$$

$$f_n' = \sqrt{\delta'^2 + \Omega_{n+l,n}^2}$$

- Rabi oscillation between the $|n, g\rangle$ and $|e, n+l\rangle$
- Side-band cooling / ground state cooling
- Single qubit operation

Resolved sideband





Un-resolved sideband

Master equation for 2-level atom in equilibrium with thermal reservoir (master eq. with spontaneous emission given by Liouvillian) :

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left[\hat{H}_{trap} + \hat{H}_{atom} + \hat{H}_I, \rho\right] + L^d\rho$$

where,

$$L^d\rho = \frac{\Gamma}{2}\left(2\sigma^-\rho'\sigma^+ - \sigma^+\sigma^-\rho - \rho\sigma^+\sigma^-\right)$$

where,

$\rho' = \frac{1}{2}\int_{-1}^{1} dz\, Y(z) e^{ik\hat{x}z}\rho e^{-ik\hat{x}z}$ where, $Y(z) = \frac{3\left(1+z^2\right)}{4}$

- Laser Doppler cooling
- Electro-magnetically induced transparency

Doppler cooling

$$V_p(x) = \frac{1}{2} m\nu^2 x^2$$ Pseudo potential (day-I)

$$v(t) = v_0 \cos(\nu t)$$ Classical velocity

$$\left(\frac{dP}{dt}\right)_{av} = F_{av} = \hbar k \Gamma \rho_{ee}$$ Rate of change of momentum

$$\rho_{ee} = \frac{\frac{s}{2}}{1+s+\left(\frac{2\delta_{eff}}{\Gamma}\right)^2}$$ excited state population

$$s = \frac{2|\Omega|^2}{\Gamma^2}$$ saturation parameter

$$\delta_{eff} = (\omega_l - \omega_{at}) - \vec{k} \cdot \vec{v}$$ effective detuning

Doppler cooling – the drag

$F_{av} = \hbar k \Gamma \rho_{ee}$ Rate of change of momentum

$F_{av} = F_0(1 + \kappa v)$ linearizing force in terms of velocity

where

$F_0 = \dfrac{\hbar k \Gamma \frac{s}{2}}{1+s+\left(\frac{2\Delta}{\Gamma}\right)^2}$ definition of force that displaces the ion

$\kappa = \dfrac{\frac{8k\Delta}{\Gamma^2}}{1+s+\left(\frac{2\Delta}{\Gamma}\right)^2}$ definition of the friction

So we have generated a viscous drag force provided $\Delta < 0$

Doppler cooling – rate and final temp.

$$\dot{E}_c = \langle F_{av} v\rangle = F_0(\langle v\rangle + \kappa\langle v^2\rangle) = F_0\kappa\langle v^2\rangle$$ cooling rate

$$\dot{E}_h = \frac{1}{2m}\frac{d}{dt}\langle P^2\rangle = \dot{E}_{abs} + \dot{E}_{emit}$$ sum of abs. and emiss. rate

$$= \dot{E}_{abs}(1+\xi) \approx \frac{1}{2m}(\hbar k)^2\Gamma\rho_{ee}(v=0)(1+\xi)$$ heating rate

$$m\langle v^2\rangle = k_B T = \frac{\hbar\Gamma}{8}(1+\xi)\left[\frac{(1+s)\Gamma}{2\Delta} + \frac{2\Delta}{\Gamma}\right]$$ final energy

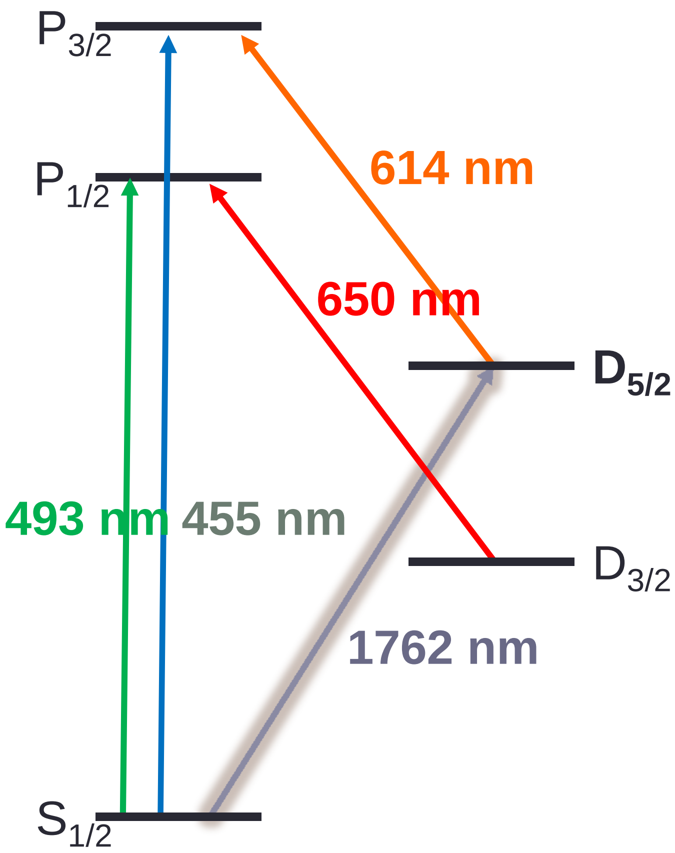
$$T_{DL} = \frac{\hbar\Gamma\sqrt{1+s}}{4k_B}(1+\xi)$$ final temperature

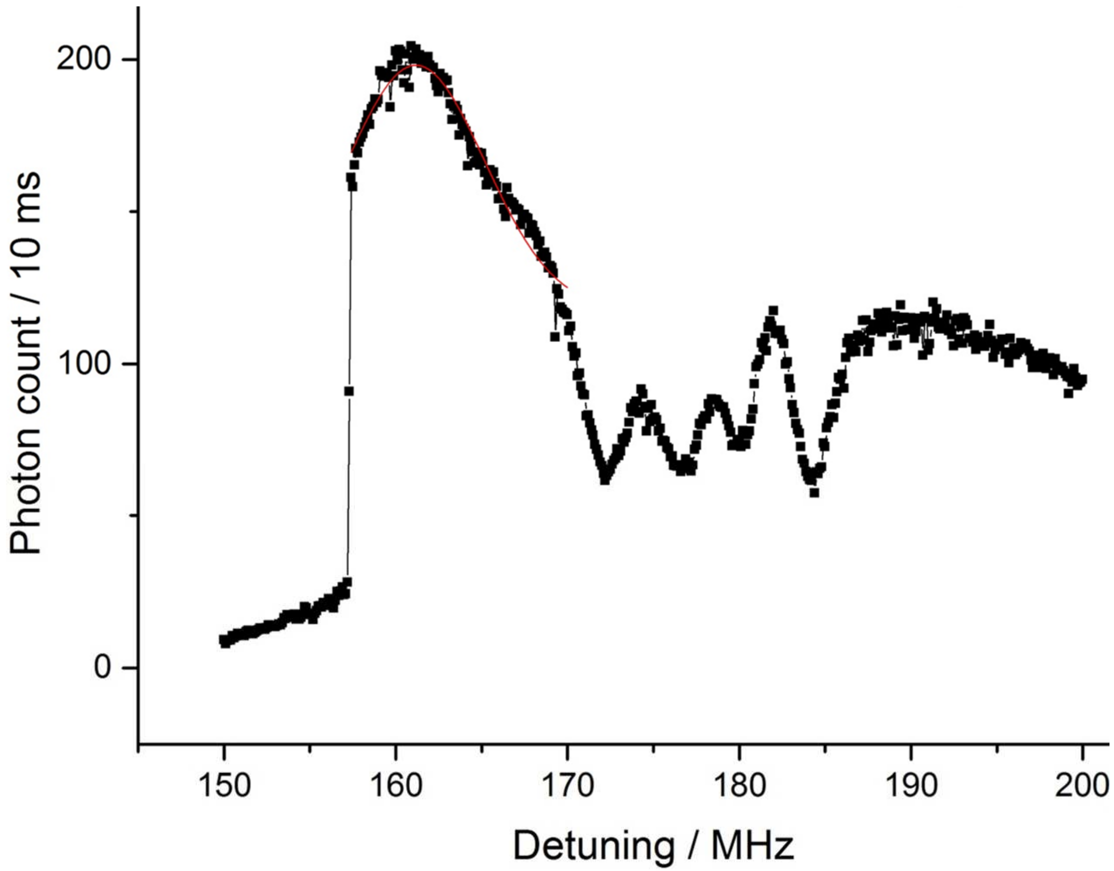
Best results

$$\Delta = \Gamma\sqrt{1+\frac{s}{2}}$$ Required detuning

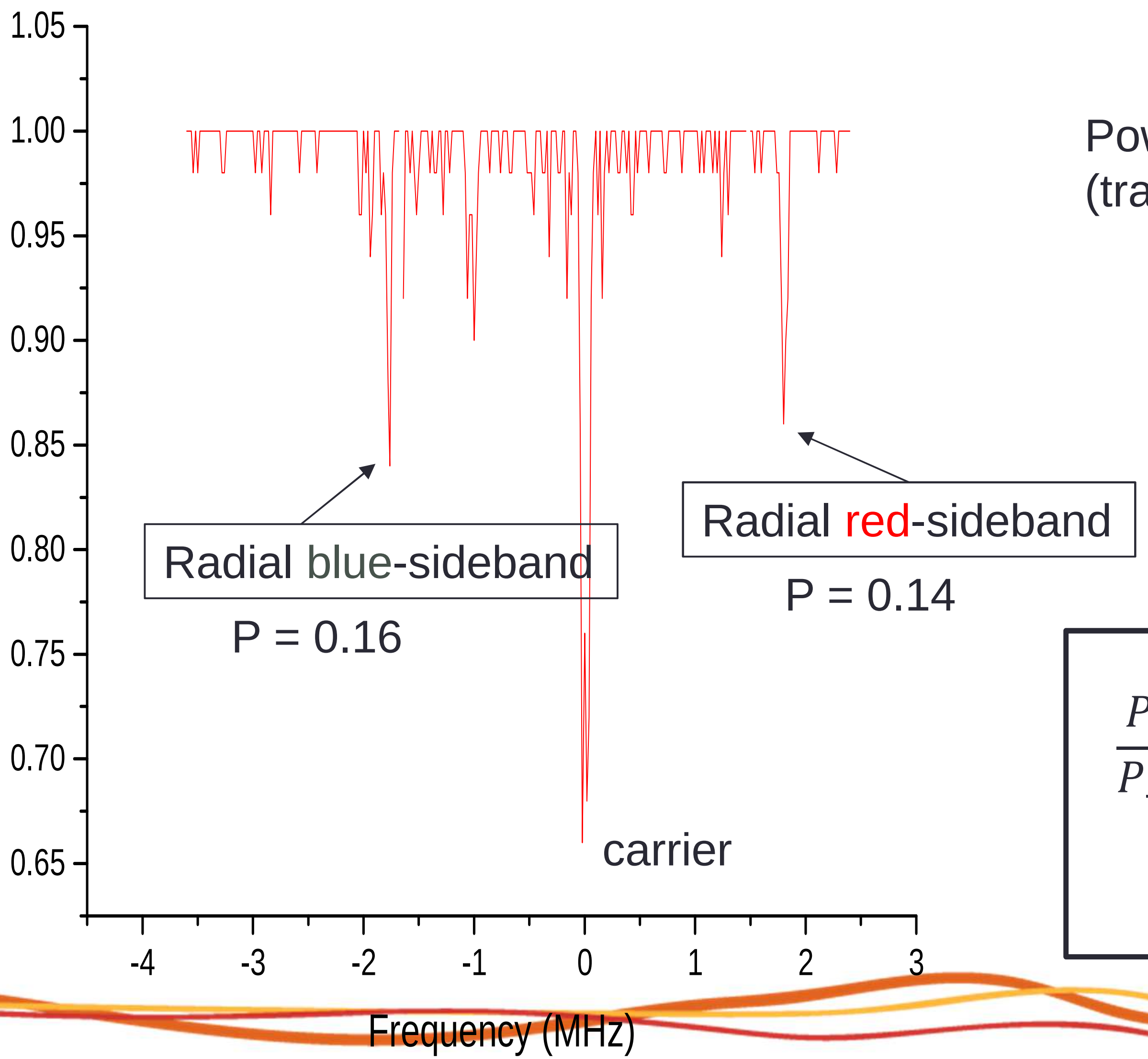
$$s = 2\frac{|\Omega|^2}{\Gamma^2}$$ saturation parameter

Doppler cooling – profile





Carrier and Side-band



Power of rf = 4 W
(trap frequency ~ 1.8 MHz)

$$\frac{P\_red}{P\_blue} = \frac{n}{n+1} = \frac{0.14}{0.16}$$

n = 7

Resolved sideband

Expanding in terms of η and keeping upto the second term

$$\hat{H}^{LD}_{inter}(t) = \frac{\hbar}{2}\Omega\left[\hat{\sigma}_{+}e^{-i\delta t} + h.c.\right]$$

Carrier with rabi frequency Ω

$$+\frac{\hbar}{2}\Omega\left\{\Sigma_{n=-\infty}^{\infty} i\eta C_{2n}\hat{\sigma}_{+}e^{-i\delta t} \times \left[\hat{a}e^{-i(\nu+n\Omega_{rf})t} + \hat{a}^{\dagger}e^{i(\nu+n\Omega_{rf})t}\right] + h.c.\right\}$$

Sidebands $\pm(\nu + n\Omega_{rf})$ with strength $\eta C_{2n}\Omega$

Condition for validity of resolved sideband

$$\Omega_{rf} \ll \nu \ll \tilde{\Gamma}$$

Sideband cooling to ground state

Adjust detuning $\delta = \omega - \omega_a = -\nu \ (n = 0)$

Carrier shifted by ν

$$H_{inter}^{LD} = \frac{\hbar}{2}\Omega[\hat{\sigma}_+ e^{i\nu t} + \hat{\sigma}_- e^{-i\nu t} + i\eta(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger\ )$$
$$+i\eta(\hat{\sigma}_+\hat{a}^\dagger e^{i2\nu t} + \hat{\sigma}_-\hat{a}e^{-i2\nu t})]$$

blue sideband at 2ν

resonant sideband

Cooling rate:

$$R_n = excited\ state\ occpancy\ probability\ (P_e(n)) \times decay\ rate$$

Sideband cooling to ground state

Cooling rate

$$R_n = excited\ state\ occpancy\ probability\ (P_e(n)) \times decay\ rate$$

$$R_n = \tilde{\Gamma} P_e(n) = \tilde{\Gamma} \frac{(\eta\sqrt{n}\Omega)^2}{2(\eta\sqrt{n}\Omega)^2 + \tilde{\Gamma}^2}$$

- The rate is depended on n
- The rate vanishes as n=0 is approached
- The final motional state is a dark state
- Dominant contribution to heating comes from carrier and 1st blue sideband absroption

Sideband cooling

Restricting to the first two motional states the rate equation (equilibrated heating and cooling):

$$\dot{P}_0 = P_1 \frac{(\eta\Omega)^2}{\tilde{\Gamma}} - P_0 \left[\left(\frac{\Omega}{2\nu}\right)^2 \tilde{\eta}^2 \tilde{\Gamma} + \left(\frac{\eta\Gamma}{4\nu}\right)^2 \tilde{\Gamma} \right]$$

$$\dot{P}_1 = -\dot{P}_0$$

P_i are probabilities to be in state $|\mathrm{i}\rangle$

In steady state $\dot{P}_i = 0$

$$\bar{n} \approx P_1 \approx \left(\frac{\tilde{\Gamma}}{2\nu}\right)^2 \left[\left(\frac{\tilde{\eta}}{\eta}\right)^2 + \frac{1}{4} \right]$$

Motional state population

$|\psi(0)\rangle = |g\rangle \Sigma_{n=0}^{\infty} c_n |n\rangle$ Initial state

$P_g(t) = \langle \psi(t) | (|g\rangle\langle g| \otimes \hat{I}_m) | \psi(t) \rangle$

probability o be in the ground state after excitation

$P_g(t) = \frac{1}{2}\left[1 + \Sigma_{n=0}^{\infty} P_n cos \Omega_{n,n+1} t\right]$ after blue sideband excitation

$P_n = |c_n|^2$ probabilities to be in motional n-state

Motional state after cooling

1. Final state is a thermal state
2. Use $P_e(t) = 1 - P_g(t)$
3. Find the probability ratio of red-to-blue sideband excitations

$$P_e^{RSB}(t) = \Sigma_{m=1}^{\infty}\left(\frac{\bar{n}}{\bar{n}+1}\right)^m \sin^2 \Omega_{m,m-1}t$$

$$= \frac{\bar{n}}{(\bar{n}+1)}\Sigma_{m=0}^{\infty}\left(\frac{\bar{n}}{\bar{n}+1}\right)^m \sin^2 \Omega_{m+1,m}t \qquad \Omega_{m+1,m} = \Omega_{m,m-1}$$

$$= \frac{\bar{n}}{\bar{n}+1}\, P_e^{BSB}(t)$$

$$R = \frac{P_e^{RSB}}{P_e^{BSB}} = \frac{\bar{n}}{\bar{n}+1}$$

Other cooling techniques

Radiative damping (applicable only to electrons in Penning traps) – classical treatment only

$$-\frac{dE}{dt} = \frac{2e^2}{3c^3}\ddot{\rho}^2$$

$$\frac{dE}{dt} = -\gamma_c E$$

$$E(t) = E_0 e^{-\gamma_c t}$$

$$\ddot{\rho} = \omega_c \times \dot{\rho}$$
$$E = \frac{1}{2}m\dot{\rho}^2$$
$$\gamma_c = \frac{4e^2\omega_c^2}{3mc^3}$$

Introducing for an electron the classical radius as $r_0 = \frac{e^2}{mc^2}$, we obtain:

$$\gamma_c = \left[\frac{4r_0\omega_c}{3c}\right]\omega_c$$

Problem 2.1.: Show that for magnetic field of 50kG, the radiative damping rate of cyclotron motion of a proton is insignificant as compared to that of an electron. Find out the scaling factor of the rate as a function of mass.

Other cooling techniques

Resistive damping – classical treatment only

Force on the charge due to image charge on the electrodes:

$$f = -\frac{e\kappa IR}{2z_0}$$

Dissipated power on the resistor

$$-\dot{z}f = I^2R$$

Therefore one obtains:

$$I = \kappa\left(\frac{e}{2z_0}\right)\dot{z}$$ Since the current is proportional to the velocity

$$\mathrm{f} = -\mathrm{m}\gamma_z\dot{z}$$ is a dissipative force

Other cooling techniques

Resistive damping – results from quantum treatment

$$\gamma_c' = \frac{4e^2\omega_+^2}{3mc^3}\frac{\omega_+}{\omega_+ - \omega_-} \text{ and } \gamma_m = \left[\frac{\omega_-}{\omega_+}\right]^3 \gamma_c'$$

Problem 2.1.:Calculate the damping rate for both modified cyclotron and magnetron motion for an electron in 50kG magnetic field. Comment on the stability of the magnetron motion.