

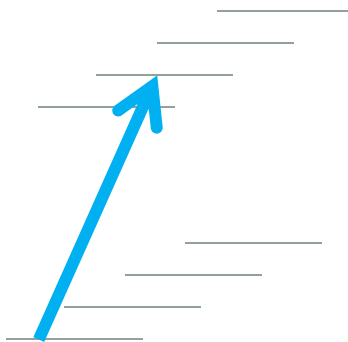
Notes to get it from

<http://coldiongroup.wixsite.com/index/manas>



Motional state population

$$|\psi(0)\rangle = |g\rangle \sum_{n=0}^{\infty} c_n |n\rangle \quad \text{Initial state}$$



$$\Omega_{n,n+1} = \Omega_0 \sqrt{n+1} \eta$$

The RSB or BSB Rabi frequency scales with n : So drive BSB or RSB and measure the probability of transfer



Motional state population

probability to be in the ground state after excitation

$$P_g(t) = \langle \psi(t) | (|g\rangle\langle g| \otimes \hat{I}_m) | \psi(t) \rangle$$

where

$$\hat{I}_m = \sum_m |m\rangle\langle m|$$

Since we already derived time evolution under RSB or BSB we get

$$P_g(t) = \frac{1}{2} \left[1 + \sum_{n=0}^{\infty} P_n \cos \Omega_{n,n+1} t \right] \quad \text{after blue sideband excitation}$$

$$P_n = |c_n|^2 \quad \text{probabilities to be in motional } n\text{-state}$$



Motional state after cooling

1. Final state is a thermal state
2. Use $P_e(t) = 1 - P_g(t)$
3. Find the probability ratio of red-to-blue sideband excitations

$$\begin{aligned}
 P_e^{RSB}(t) &= \sum_{m=1}^{\infty} \left(\frac{\bar{n}}{\bar{n}+1} \right)^m \sin^2 \Omega_{m,m-1} t \\
 &= \frac{\bar{n}}{(\bar{n}+1)} \sum_{m=0}^{\infty} \left(\frac{\bar{n}}{\bar{n}+1} \right)^m \sin^2 \Omega_{m+1,m} t \\
 &= \frac{\bar{n}}{\bar{n}+1} P_e^{BSB}(t)
 \end{aligned}
 \qquad \Omega_{m+1,m} = \Omega_{m,m-1}$$

$$R = \frac{P_e^{RSB}}{P_e^{BSB}} = \frac{\bar{n}}{\bar{n} + 1}$$



Other cooling techniques

Radiative damping (applicable only to electrons in Penning traps) – classical treatment only

$$-\frac{dE}{dt} = \frac{2e^2}{3c^3} \ddot{\rho}^2$$

$$\frac{dE}{dt} = -\gamma_c E$$

$$E(t) = E_0 e^{-\gamma_c t}$$

$$\begin{aligned} \ddot{\rho} &= \omega_c \times \dot{\rho} \\ E &= \frac{1}{2} m \dot{\rho}^2 \\ \gamma_c &= \frac{4e^2 \omega_c^2}{3mc^3} \end{aligned}$$

Introducing for an electron the classical radius as $r_0 = \frac{e^2}{mc^2}$, we obtain:

$$\gamma_c = \left[\frac{4r_0 \omega_c}{3c} \right] \omega_c$$

Problem 2.1.: Show that for magnetic field of 50kG, the radiative damping rate of cyclotron motion of a proton is insignificant as compared to that of an electron. Find out the scaling factor of the rate as a function of mass.

Other cooling techniques

Resistive damping – classical treatment only

Force on the charge due to image charge on the electrodes:

$$f = -\frac{e\kappa IR}{2z_0}$$

Dissipated power on the resistor

$$-\dot{z}f = I^2 R$$

Therefore one obtains:

$$I = \kappa \left(\frac{e}{2z_0} \right) \dot{z} \quad \text{Since the current is proportional to the velocity}$$

$$f = -m\gamma_z \dot{z} \quad \text{is a dissipative force}$$



Other cooling techniques

Resistive damping – results from quantum treatment

$$\gamma'_c = \frac{4e^2\omega_+^2}{3mc^3} \frac{\omega_+}{\omega_+ - \omega_-} \quad \text{and} \quad \gamma_m = \left[\frac{\omega_-}{\omega_+} \right]^3 \gamma'_c$$

Problem 2.1.: Calculate the damping rate for both modified cyclotron and magnetron motion for an electron in 50kG magnetic field. Comment on the stability of the magnetron motion.



Ion chain and modes

Potential for a chain of ions under assumptions:

1. Strong radial confinement and weak axial (x) confinement
2. Negligible micromotion

$$V = \sum_{m=1}^N \frac{1}{2} M \nu^2 x_m(t)^2 + \sum_{m,n=1, m \neq n}^N \frac{Z^2 e^2}{8\pi\epsilon_0} \frac{1}{|x_n(t) - x_m(t)|}$$

$$x_m(t) \approx x_m^{(0)} + q_m(t)$$

ν : angular secular frequency

M : Mass of the ion

$x_m^{(0)}$: equilibrium position of the ion



Ion chain and modes

The ion's equilibrium will be decided by:

$$\left[\frac{\partial V}{\partial x_m} \right]_{x_m = x_m^{(0)}} = 0$$

Redefine a new length scale as :

$$l^3 = \frac{Z^2 e^2}{4\pi\epsilon_0 M v^2}$$

Rescaled equilibrium will be :

$$u_m = \frac{x_m^{(0)}}{l}$$



Ion chain and modes

So we obtain coupled algebraic equations as:

$$u_m - \sum_{n=1}^{m-1} \frac{1}{(u_m - u_n)^2} + \sum_{n=m+1}^N \frac{1}{(u_m - u_n)^2} = 0$$

$$(m = 1, 2, \dots, N)$$

Only for small number analytic solution is possible like:

$$N = 2: \quad u_1 = -\left(\frac{1}{2}\right)^{\frac{2}{3}}, \quad u_2 = \left(\frac{1}{2}\right)^{\frac{2}{3}}$$

$$N = 3: \quad u_1 = -\left(\frac{5}{4}\right)^{\frac{1}{3}}, \quad u_2 = 0, \quad u_3 = \left(\frac{5}{4}\right)^{\frac{1}{3}}$$



Ion chain and modes

Otherwise numerical solutions for higher number:

N	Scaled equilibrium positions									
2										
3										
4										
5										
6										
7										
8										
9										
10										

Important to note the minimum spacing occurs near the center and it obeys empirical relation as

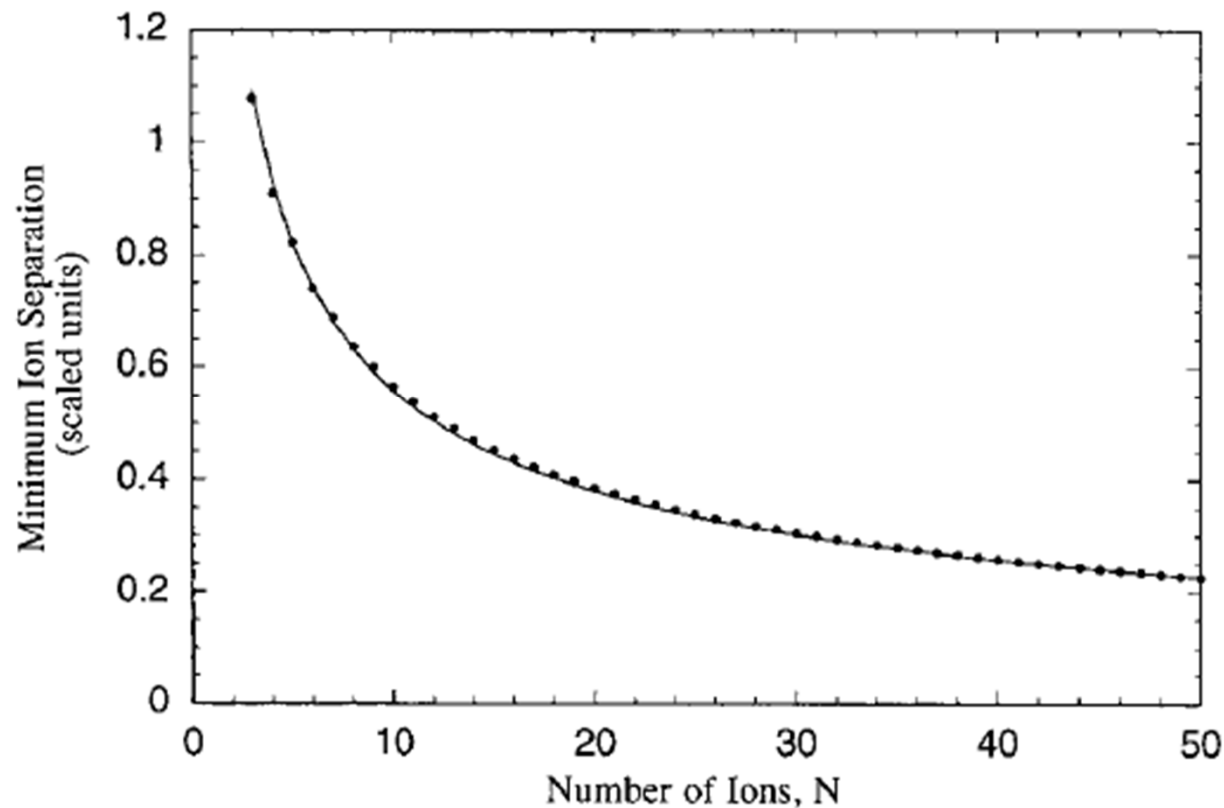
$$u_{min}(N) \approx \frac{2.018}{N^{0.559}}$$

$$x_{min}(N) = \left(\frac{Z^2 e^2}{4\pi\epsilon_0 M v^2} \right)^{\frac{1}{3}} \frac{2.018}{N^{0.559}}$$



Ion chain and modes

For larger numbers numerical results provide:



$$u_{\min}(N) \approx \frac{2.018}{N^{0.559}}$$



Ion chain and modes

Quantum fluctuations:

The Lagrangian:

$$L = \frac{M}{2} \sum_{m=1}^N (\dot{q}_m)^2 - \frac{1}{2} \sum_{n,m=1}^N q_n q_m \left[\frac{\partial^2 V}{\partial x_n \partial x_m} \right]_0$$

The derivative should be taken at $q_{n,m} = 0$ and $O(q_n^3)$ neglected

More explicitly:

$$L = \frac{M}{2} \left[\sum_{m=1}^N (\dot{q}_m)^2 - v^2 \sum_{n,m=1}^N A_{nm} q_n q_m \right]$$

where

$$A_{nm} = -\frac{2}{|u_m - u_n|^3} \quad n \neq m$$

$$A_{nm} = 1 + 2 \sum_{p=1, \neq m}^N \frac{1}{|u_m - u_n|^3} \quad n = m$$



Ion chain and modes

Since matrix A is real, symmetric non-negative and definite, the eigenvalues are therefore non-negative. The eigenvectors are:

$$\sum_{n=1}^N A_{nm} b_n^{(p)} = \mu_p b_m^{(p)} \quad (\text{where } p = 1, \dots, N) \text{ and } \mu_p \geq 0$$

The eigenvectors are ordered in increasing order of eigen values.
The eigenvectors are also normalized as:

$$\sum_{p=1}^N b_n^{(p)} b_m^{(p)} = \delta_{nm}$$
$$\sum_{n=1}^N b_n^{(p)} b_n^{(q)} = \delta_{pq}$$



Ion chain and modes

The first and second eigenvectors may be evaluated as:

$$b^{(1)} = \frac{1}{\sqrt{N}} \{1, 1, \dots, 1\}, \quad \mu_1 = 1$$

$$b^{(2)} = \frac{1}{\sqrt{\sum_{m=1}^N u_m^2}} \{u_1, u_2, \dots, u_N\}, \quad \mu_2 = 3$$

Thus for two/three ions:

$$N = 2: \quad b^{(1)} = \frac{1}{\sqrt{2}} (1, 1), \quad \mu_1 = 1$$

$$b^{(2)} = \frac{1}{\sqrt{2}} (-1, 1), \quad \mu_2 = 3$$

$$N = 3: \quad b^{(1)} = \frac{1}{\sqrt{3}} (1, 1, 1), \quad \mu_1 = 1$$

$$b^{(2)} = \frac{1}{\sqrt{2}} (-1, 0, 1), \quad \mu_2 = 3$$

$$b^{(3)} = \frac{1}{\sqrt{2}} (1, -2, 1), \quad \mu_3 = \frac{29}{5}$$



Ion chain and modes

Thus the normal modes of the ion motion are defined as :

$$Q_p(t) = \sum_{m=1}^N b_m^{(p)} q_m(t)$$

Thus the Lagrangian is:

$$L = \frac{M}{2} \sum_{p=1}^N \left[\dot{Q}_p^2 - v_p^2 Q_p^2 \right]$$

With

$$v_p = \sqrt{\mu_p} \nu$$



Ion chain and modes

Thus the Hamiltonian becomes:

$$\hat{H} = \frac{1}{2M} \sum_{p=1}^N P_p^2 + \frac{M}{2} \sum_{p=1}^N \nu_p^2 Q_p^2$$

Solving the same way as before:

$$\begin{aligned} \widehat{q_m}(t) &= \sum_{p=1}^N b_m^{(p)} \widehat{Q_p}(t) \\ &= i \sqrt{\frac{\hbar}{2M\nu N}} \sum_{p=1}^N s_m^{(p)} (\widehat{a_p} e^{-i\nu_p t} - \widehat{a_p}^T e^{i\nu_p t}) \end{aligned}$$

The coupling is:

$$s_m^{(p)} = \frac{\sqrt{N} b_m^{(p)}}{\mu_p^{\frac{1}{4}}}$$



Ion chain and modes

The coupling is:

$$s_m^{(p)} = \frac{\sqrt{N} b_m^{(p)}}{\mu_p^{\frac{1}{4}}}$$

Example: COM

$$s_m^{(1)} = 1$$

$$v_1 = v$$

Example: BM

$$s_m^{(2)} = \frac{\sqrt{N}}{3^{\frac{1}{4}}} \frac{1}{(\sum_{m=1}^N u_m^2)^{\frac{1}{2}}} u_m$$

$$v_2 = \sqrt{3}v$$



Special motional states

In Heisenberg picture the C operators as defined before are time independent and corresponds to annihilation operator

$$\hat{N} = \hat{C}^T(t)\hat{C}(t) = \hat{a}^T \hat{a}$$

$$\hat{a}|n\rangle_v = \sqrt{n}|n-1\rangle_v$$

$$\hat{a}^T|n\rangle_v = \sqrt{n+1}|n+1\rangle_v$$

$$\hat{N}|n\rangle_v = n|n\rangle_v$$

In Schroedinger's picture

$$\hat{U}^T(t)\hat{N}\hat{U}(t) = \hat{C}_S^T(t)\hat{C}_S(t)$$



Special motional states

Now in SP the operators and states are:

$$\hat{C}_S(t)|n, t\rangle = \sqrt{n} |n - 1, t\rangle$$

$$\hat{C}_S^T(t)|n, t\rangle = \sqrt{n + 1} |n + 1, t\rangle$$

$$|n, t\rangle = \frac{\hat{C}_S^T(t)}{\sqrt{n!}} |n = 0, t\rangle$$

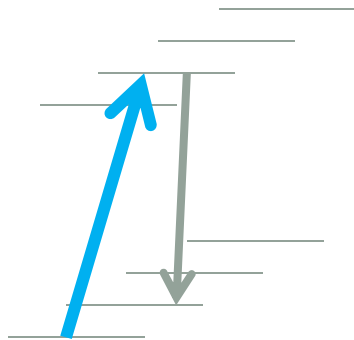
This can be used in analogy to static harmonic potential with eigen states as Fock states but these are not the energy eigenvalues.

So any motional state can be written as

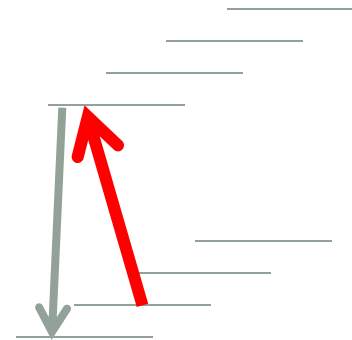
$$|\psi(t)\rangle = \sum_0^\infty c_n |n, t\rangle$$



Special motional states (Fock-state)



$$\Omega_{n,n+1} = \Omega_0 \sqrt{n+1} \eta \quad \begin{array}{l} \pi - \text{pulse} \\ C\pi - \text{pulse} \end{array}$$



$$\Omega_{n,n-1} = \Omega_0 \sqrt{n} \eta \quad \begin{array}{l} \pi - \text{pulse} \\ C\pi - \text{pulse} \end{array}$$

Thus arbitrary Fock state can be prepared

$$|\psi(t)\rangle = \sum_0^\infty c_n |n, t\rangle$$



Special motional states (coherent state)

Eigen states of annihilation operator are the coherent state:

$$\hat{C}_S(t)|\alpha\rangle = \alpha|\alpha\rangle$$

Using Fock's basis expansion:

$$|\alpha\rangle = \sum_0^\infty c_n |n\rangle$$

$$c_n = \frac{\alpha^n}{\sqrt{n!}} e^{-\frac{|\alpha|^2}{2}}$$

The probability distribution in number basis is

$$P_n = |c_n|^2 = |\langle n|\alpha\rangle|^2 = \frac{\bar{n}^n e^{-\bar{n}}}{n!} \quad \text{with } \bar{n} = |\alpha|^2$$



Special motional states (coherent state)

Eigen states of annihilation operator are the coherent state:

$$\hat{C}_S(t)|\alpha\rangle = \alpha|\alpha\rangle$$

How to generate such a state?

$$\hat{D}(\alpha) = e^{\alpha\hat{C}_S^T(t) - \alpha^*\hat{C}_S(t)}|0\rangle = |\alpha\rangle \quad \text{Apply Displacement operator}$$

To see it displaces perform similarity transformation

$$\hat{D}^T(\alpha)\hat{a}\hat{D}(\alpha) = \hat{a} + \alpha$$

$$\hat{D}(\alpha)\hat{a}\hat{D}^T(\alpha) = \hat{a} - \alpha$$

To get coherent state of phonons just shake the trap



Special motional states (thermal state)

If the ion is in thermal equilibrium with external heat bath at T the n -th state will have weightage:

$$w_n = e^{-\frac{n\hbar\nu}{k_B T}}$$

It only make sense by average and the average n provides a value of the equilibrium temperature

$$T = \frac{\hbar\nu}{k_B \ln\left(\frac{\bar{n} + 1}{\bar{n}}\right)}$$



Special motional states (thermal state)

How to write a thermal state?

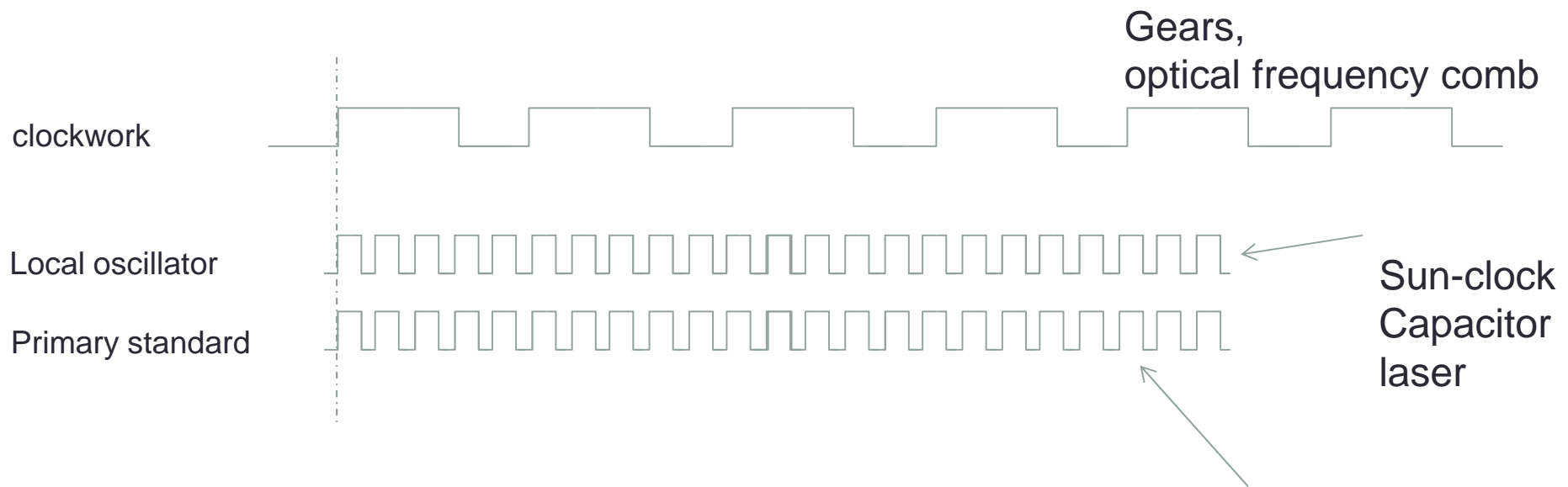
$$\rho_{th} = \frac{1}{\bar{n} + 1} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{\bar{n} + 1} \right)^n |n\rangle\langle n|$$

Measurement of these probabilities are performed by adiabatic passage protocols



Atomic clock: RF trap

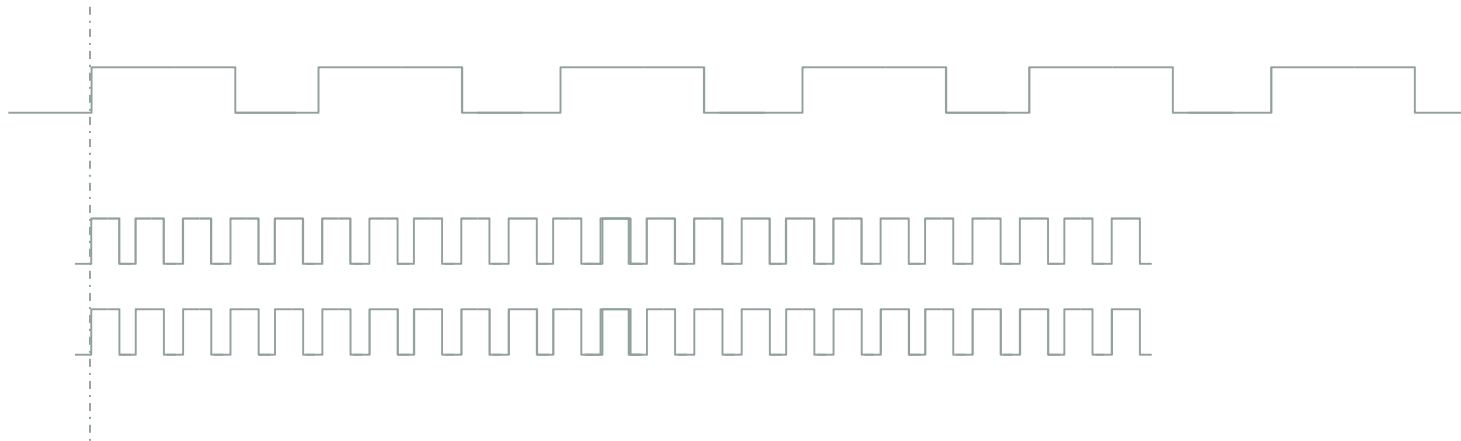
Synchronization of local oscillator to a standard frequency generator



Sun, quartz, atom



Atomic clock: RF trap



- Sun: Duration of day/night varies with season
- Quartz: strong temperature dependence, varies from sample to sample
- Atoms: universal and fundamental



Atomic clock: RF trap

Figures of merit for a good clock standard:

- Stability
- High frequency
- Low systematic uncertainties



Atomic clock: RF trap

Figures of merit for a good clock standard:

Stability

$$\delta y_1 = \frac{\delta f}{f_0} = \frac{\delta S}{f_0} \frac{1}{\frac{dS}{df}} = \frac{\delta S}{S_0 Q \kappa_S}$$

$$Q = \frac{f_0}{\Delta f}$$

$$\kappa_S = \left(\frac{dS}{df} \right) \frac{\Delta f}{S_0} \sim 1$$



Atomic clock: RF trap

Stability

for a single feedback cycle:

$$\delta y_1 = \frac{\delta f}{f_{01}} = \frac{\delta S}{f_0} \frac{1}{\frac{dS}{df}} = \frac{\delta S}{S_0 Q \kappa_S}$$

The important parameter is SNR



Atomic clock: RF trap

Stability

for a single feedback cycle: $\delta y_1 = \frac{\delta f}{f_0} = \frac{\delta S}{f_0} \frac{1}{\frac{dS}{df}} = \frac{\delta S}{S_0 Q \kappa_S}$

Therefore

$$\sigma_y(\tau) = \left(\frac{\delta f}{f_0} \right)_1 \sqrt{\frac{1}{M}} = \frac{\delta S}{f_0 \left(\frac{dS}{df} \right)} \sqrt{\frac{T_m}{\tau}} = \frac{\delta S}{S_0 Q \kappa_S} \sqrt{\frac{T_m}{\tau}}$$



Atomic clock: RF trap

High frequency

$$\sigma_y(\tau) = \frac{\delta S}{S_0 Q \kappa_S} \sqrt{\frac{T_m}{\tau}}$$

The quality factor $Q = \frac{f_0}{\delta f}$ for optical transitions it is about 10^{18} as compared to 10^{12} for microwave.

So candidates are: atoms and ions with transitions in UV



Atomic clock: RF trap

Systematic uncertainties

1. Environmental perturbation
 1. Electric field
 2. Magnetic field
2. Relativistic shifts
 1. Doppler shift
 2. Gravitational redshift



Atomic clock: RF trap

Systematic uncertainties

Electric field

Electric quadrupole shift due to the Hamiltonian $H_Q = \nabla E \cdot Q = \frac{\partial E_i}{\partial x_j} Q_{ij}$

The gradient is due to the trapping potential: $\phi(x, y, z) = A[(1 + \epsilon)x^2 + (1 - \epsilon)y^2 - 2z^2]$

Magnetic field

Usually the magnetic field applied is weak and hence the effects are much less

Black Body Radiation (BBR) shift

This is AC Stark shift due to BBR at room temperature (usually)



Atomic clock: RF trap

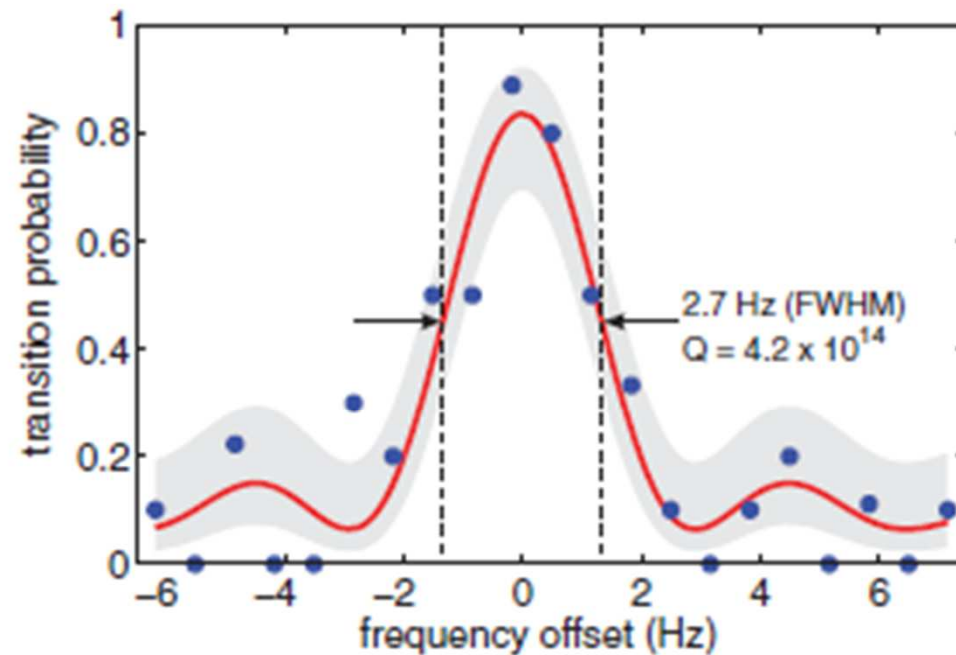


FIG. 7 (color online). Resonance of the Al^+ clock transition using quantum logic spectroscopy. From Chou, Hume, Rosenband, and Wineland, 2010.



PENNING TRAP EXPERIMENTS

Content

1. Precision mass measurements
2. G-factor measurement
3. Other possibilities



Mass measurement using PT



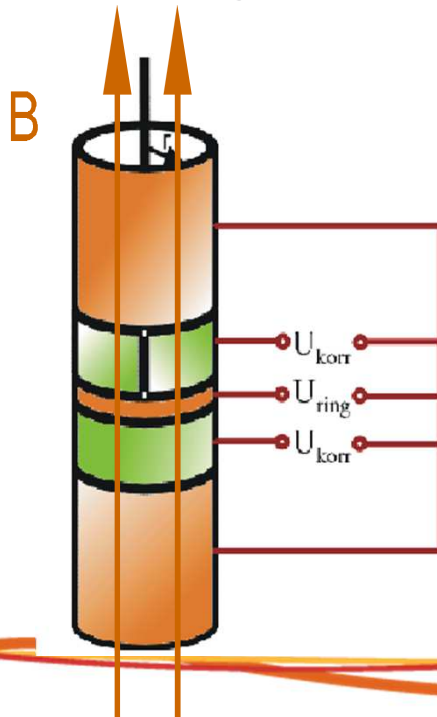
Frans Michel
Penning



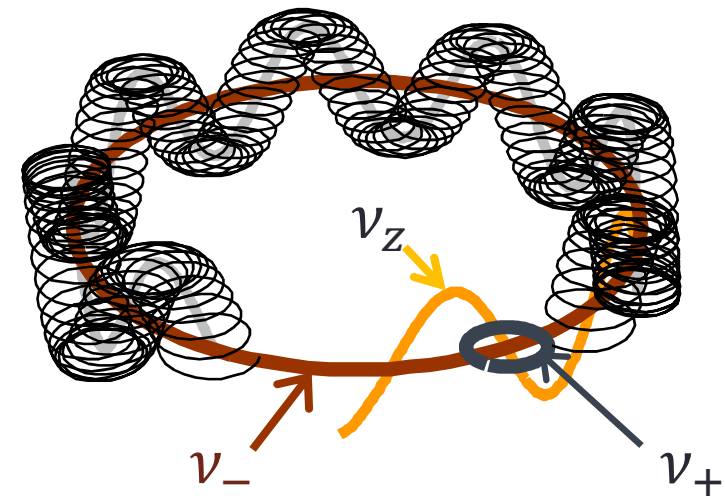
Wolfgang
Paul



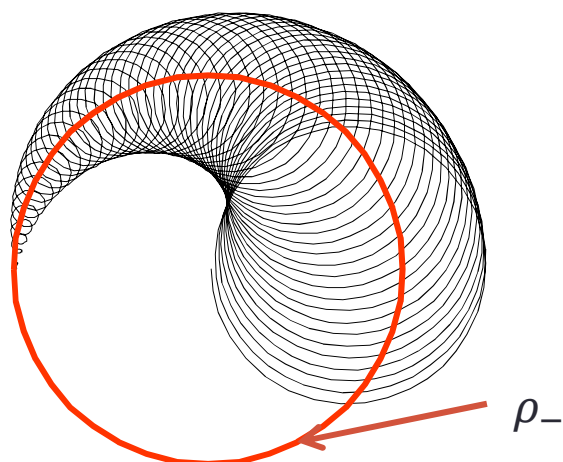
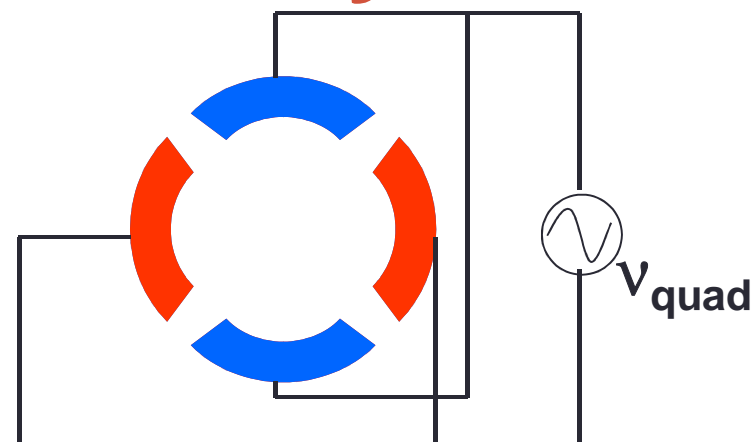
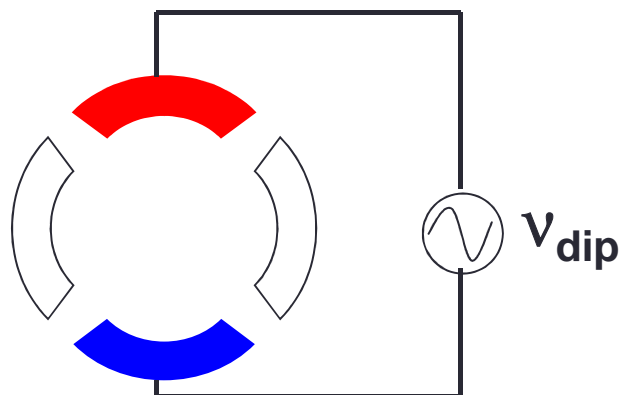
H.-Jürgen Kluge



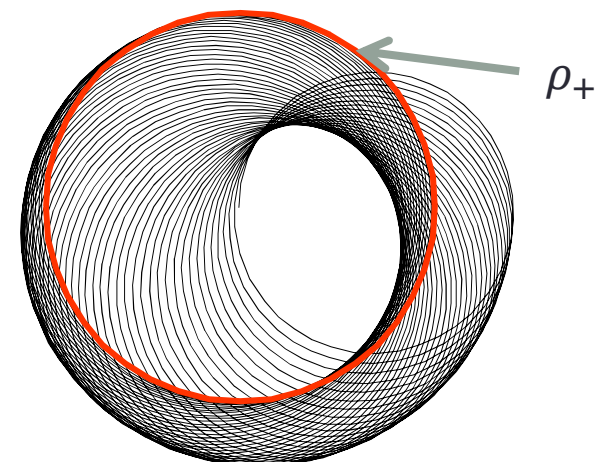
$$\nu_c = \frac{1}{2\pi} \frac{e}{m} B$$



TOF-ICR mass spectrometry



Dipole excitation: magnetron motion



Quadrupole excitation: cyclotron motion

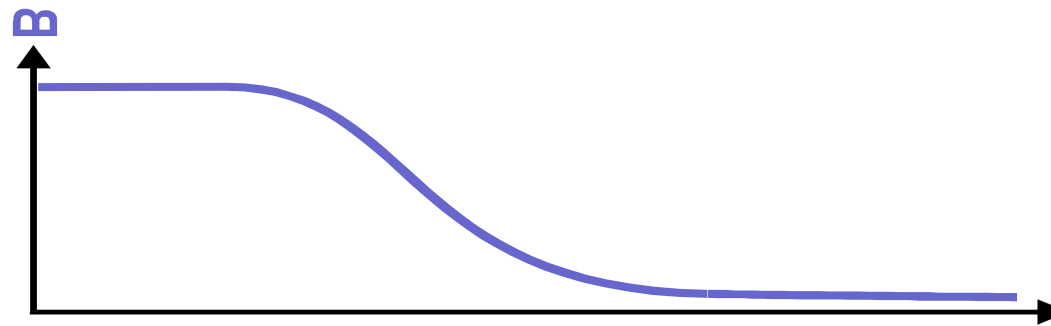


TOF-ICR mass spectrometry

Trap

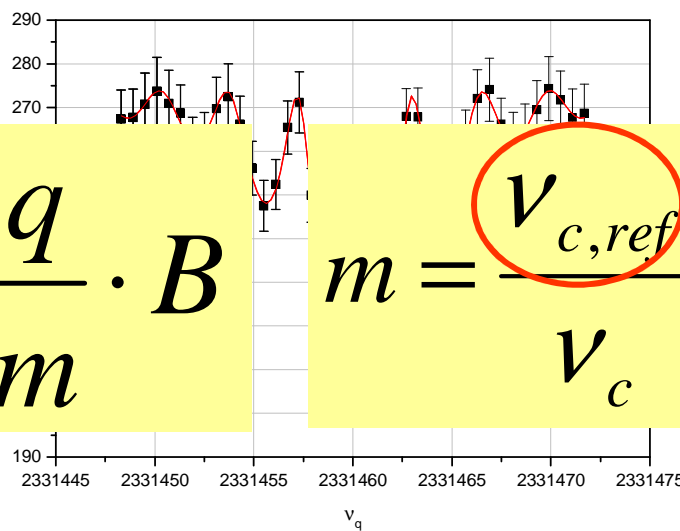
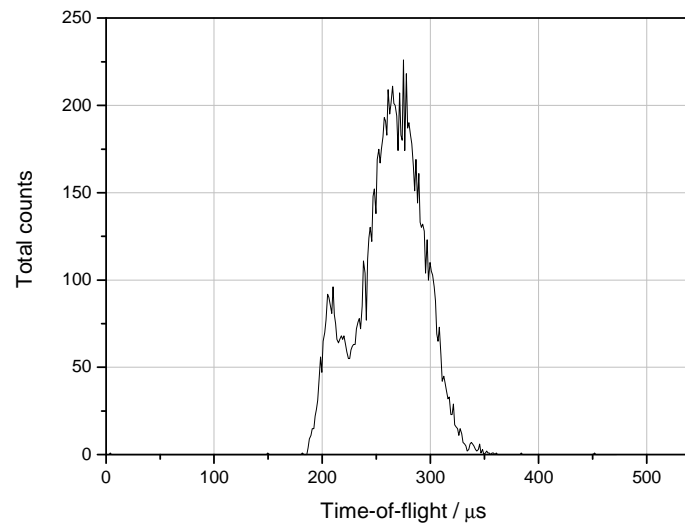
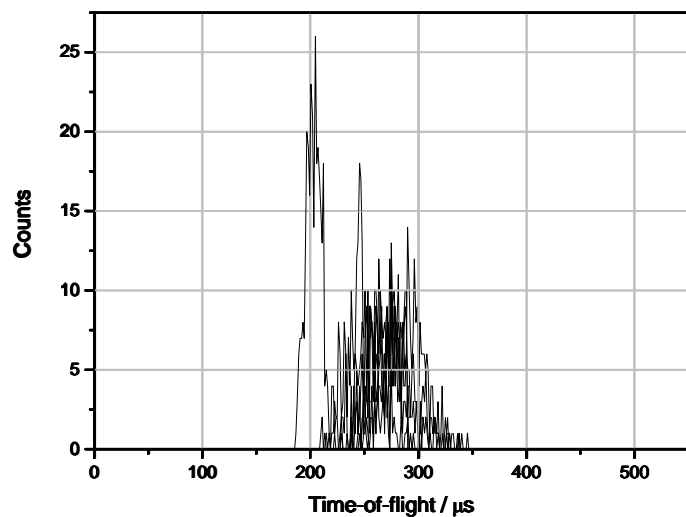
Drift section

Detector



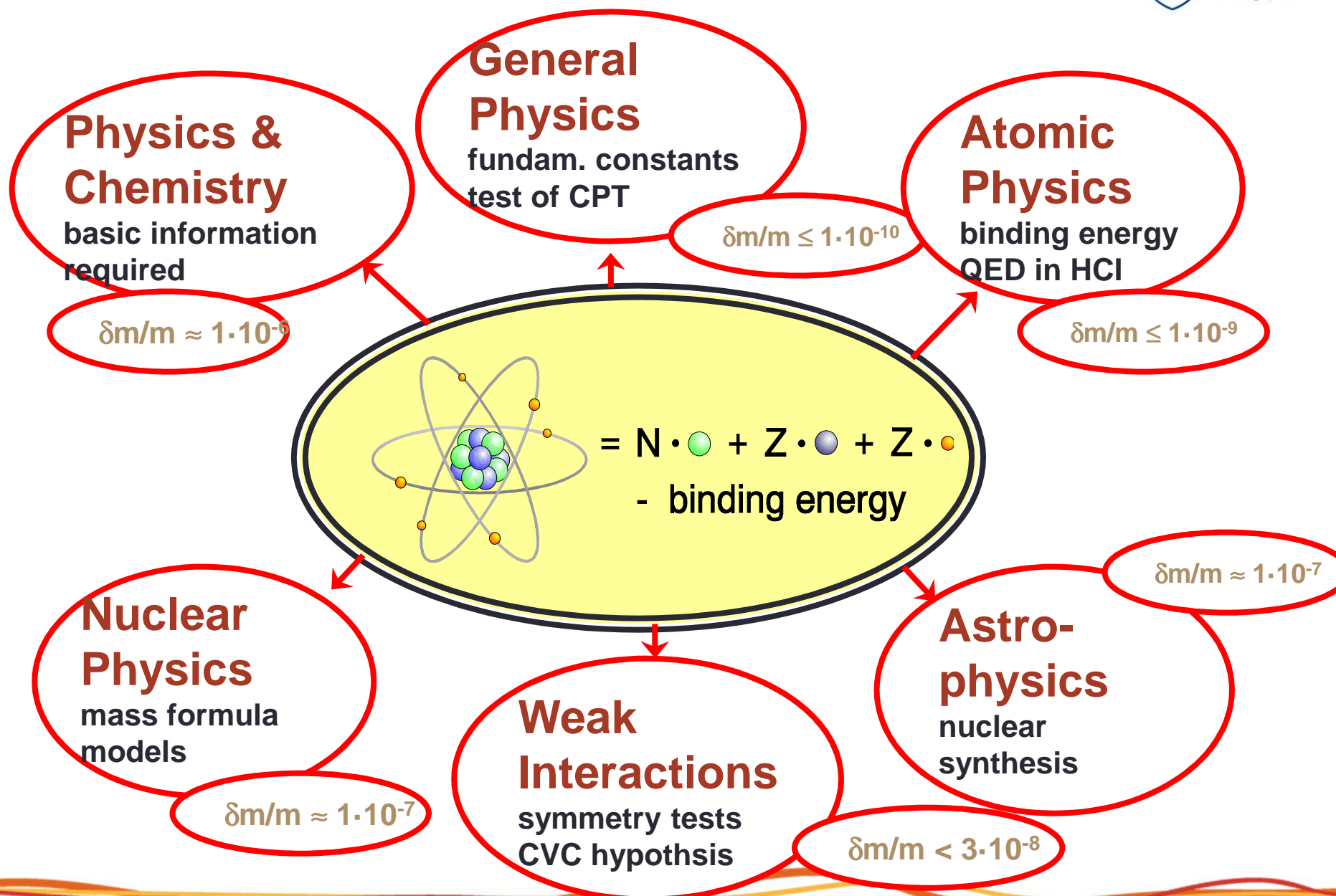
$$F_z = \bar{\mu} \frac{\partial \vec{B}}{\partial z}$$



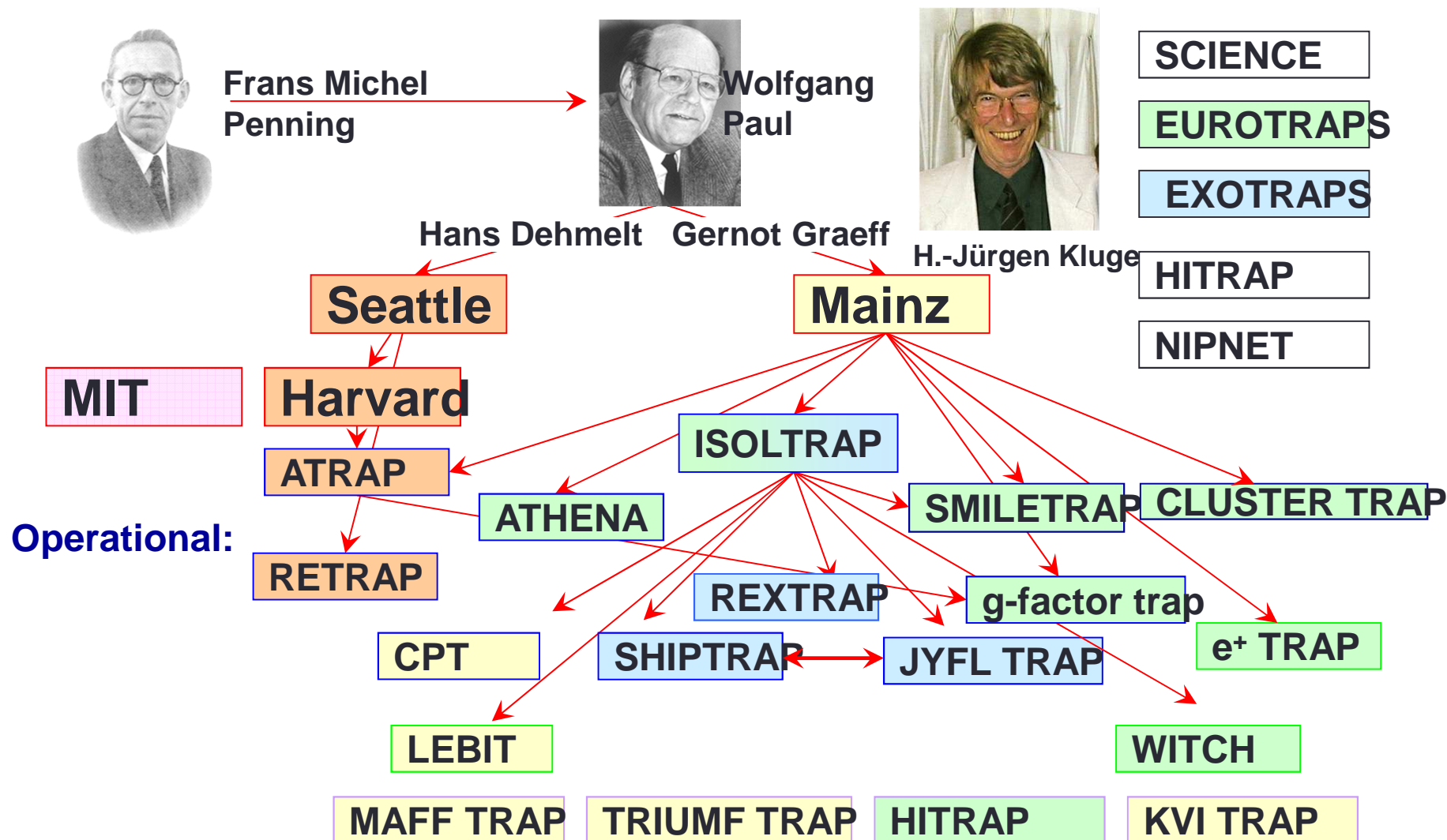


$$v_c = \frac{1}{2\pi} \cdot \frac{q}{m} \cdot B$$

$$m = \frac{v_{c,ref}}{v_c} (m_{ref} - m_e) + m_e$$



Where does it come from?



Anti-H: PT

Aim is to test CPT theorem:

Under simultaneous transformation of C, P and T the laws of physics remains the same / invariant.

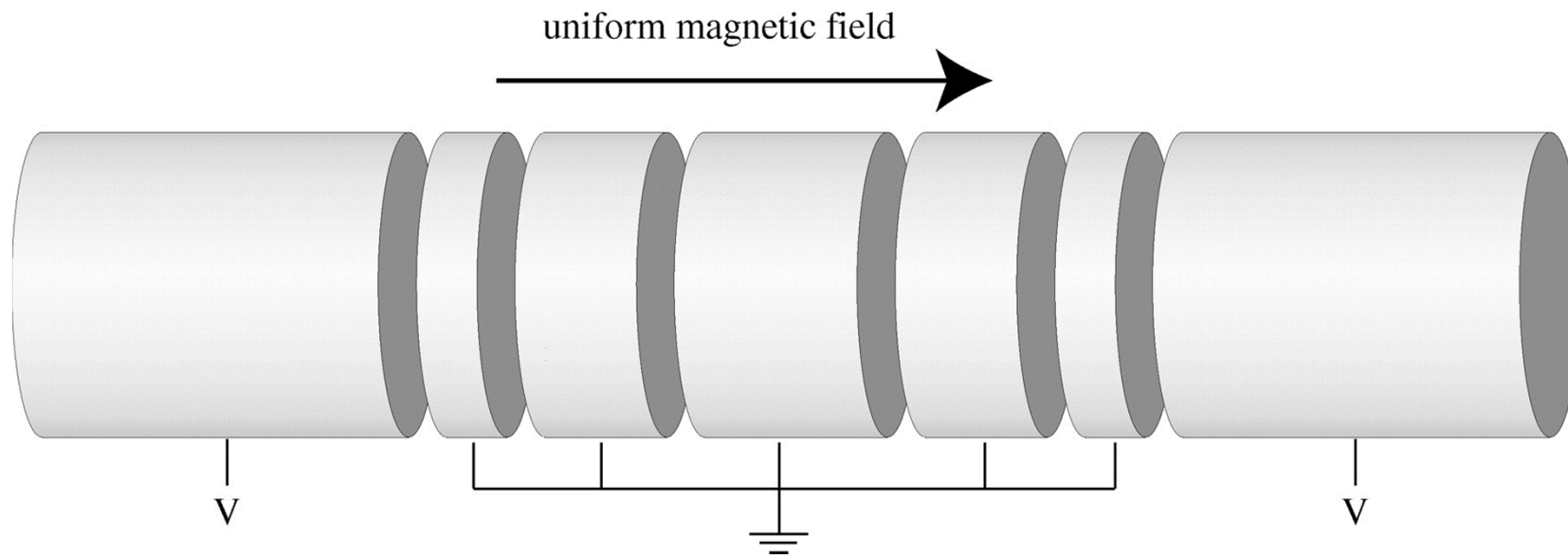
any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian must have CPT symmetry

"the feature of nature that says experimental results are independent of the orientation or the boost velocity of the laboratory through space".

N. Madsen DOI: 10.1098/rsta.2010.0026



Anti-H: PT



Penning trap with nested trap potential

N. Madsen DOI: 10.1098/rsta.2010.0026



Anti-H: PT

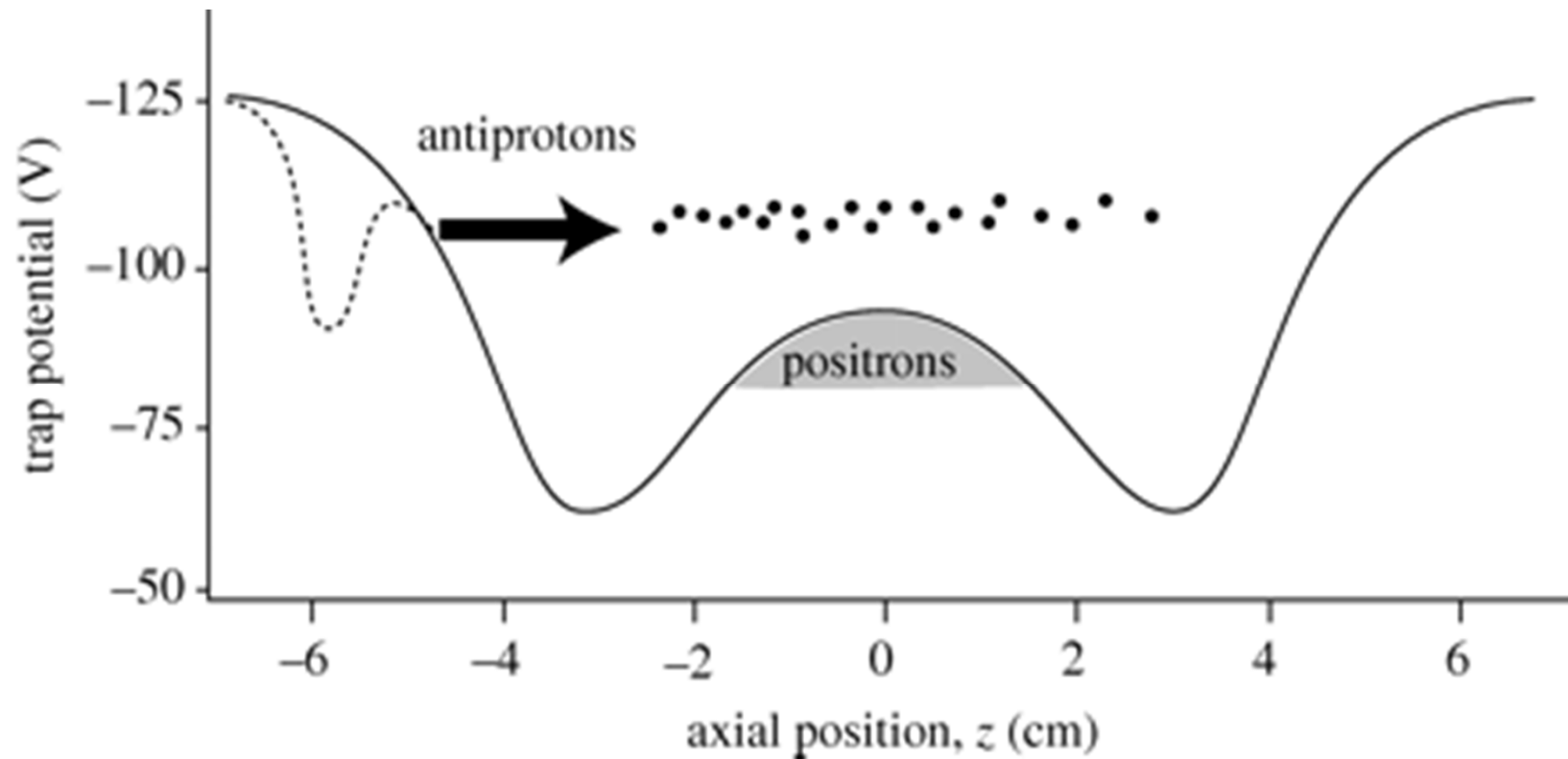
Steps for anti-hydrogen production

1. Anti-proton production from 26 GeV 10^{13} protons
2. Produce $\sim 30 \times 10^6$ anti-protons at 5 MeV @ 2 min
3. Foil to decelerate 5×10^3 are usable $\sim keV$
4. Loading anti-protons to Penning trap
5. Cooling with electrons – sympathetic cooling
6. Accumulation of positron from sodium decay
7. Production of anti-hydrogen

N. Madsen DOI: 10.1098/rsta.2010.0026



Anti-H: PT

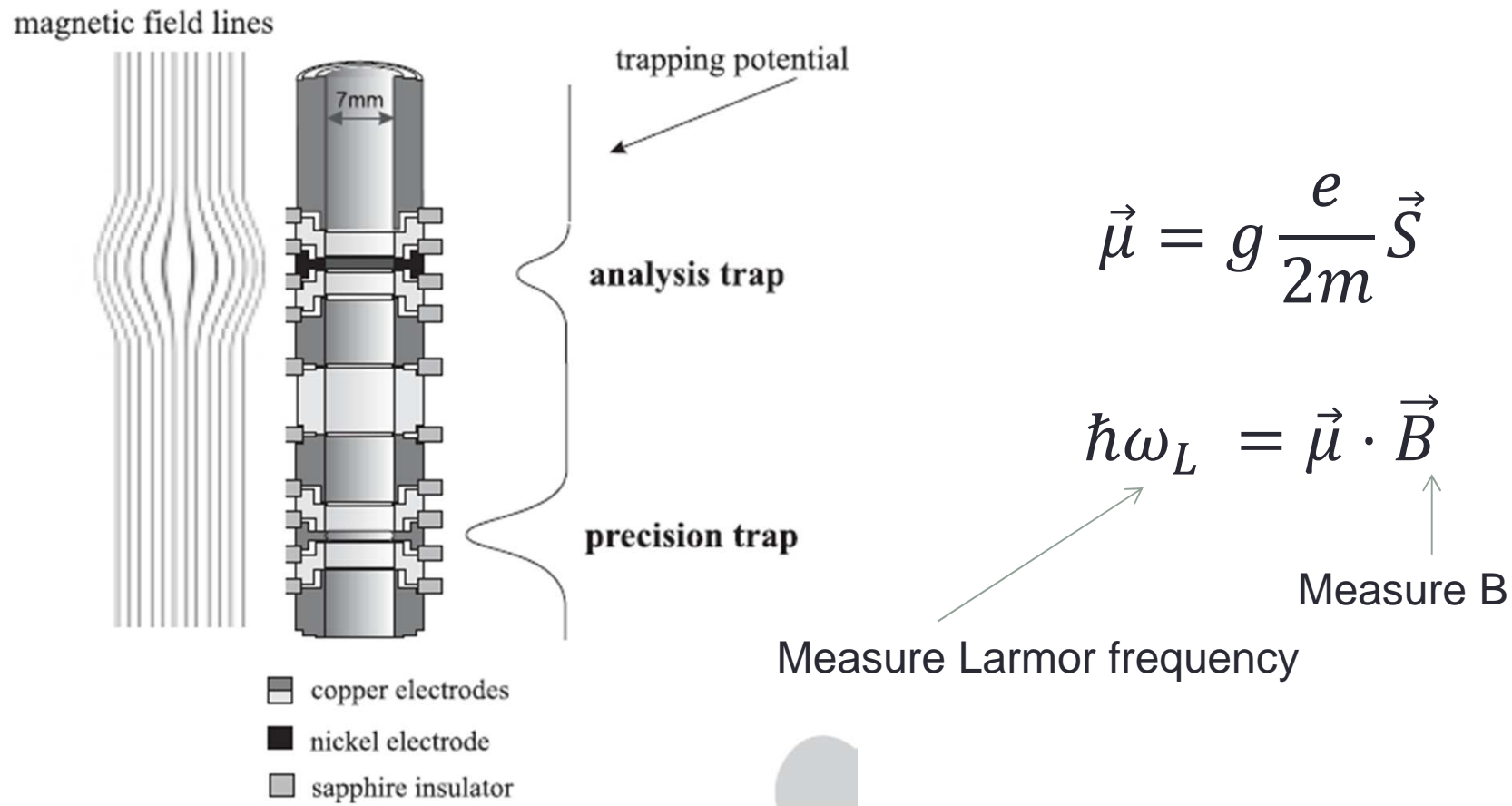


Penning trap with nested trap potential

N. Madsen DOI: 10.1098/rsta.2010.0026



g-factor measurements: PT

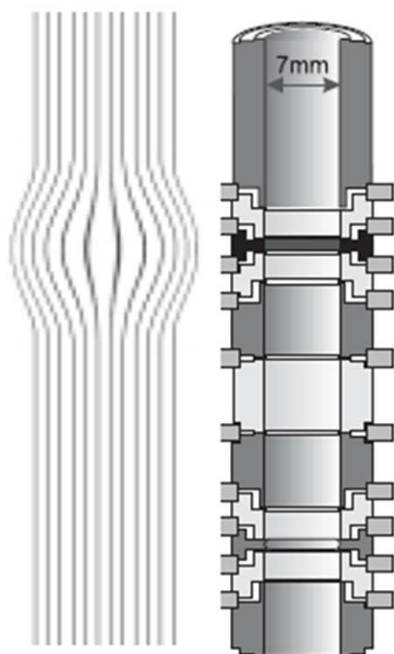


G. Werth et al. Adv. In Atom. Mol. Opt. Phys. 48, 191

G-factor measurements: PT

Continuous Stern-Gerlach experiment




magnetic field lines



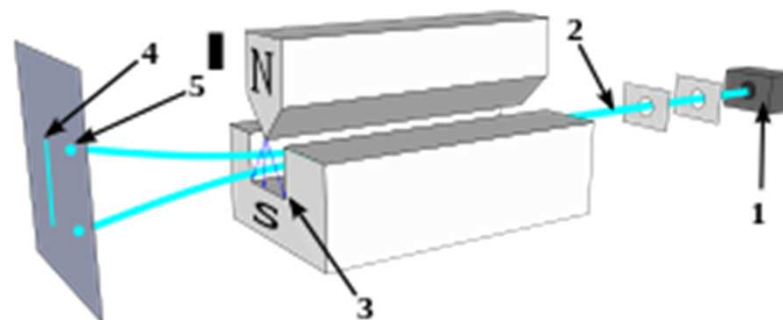
trapping potential

analysis trap

precision trap

-  copper electrodes
-  nickel electrode
-  sapphire insulator

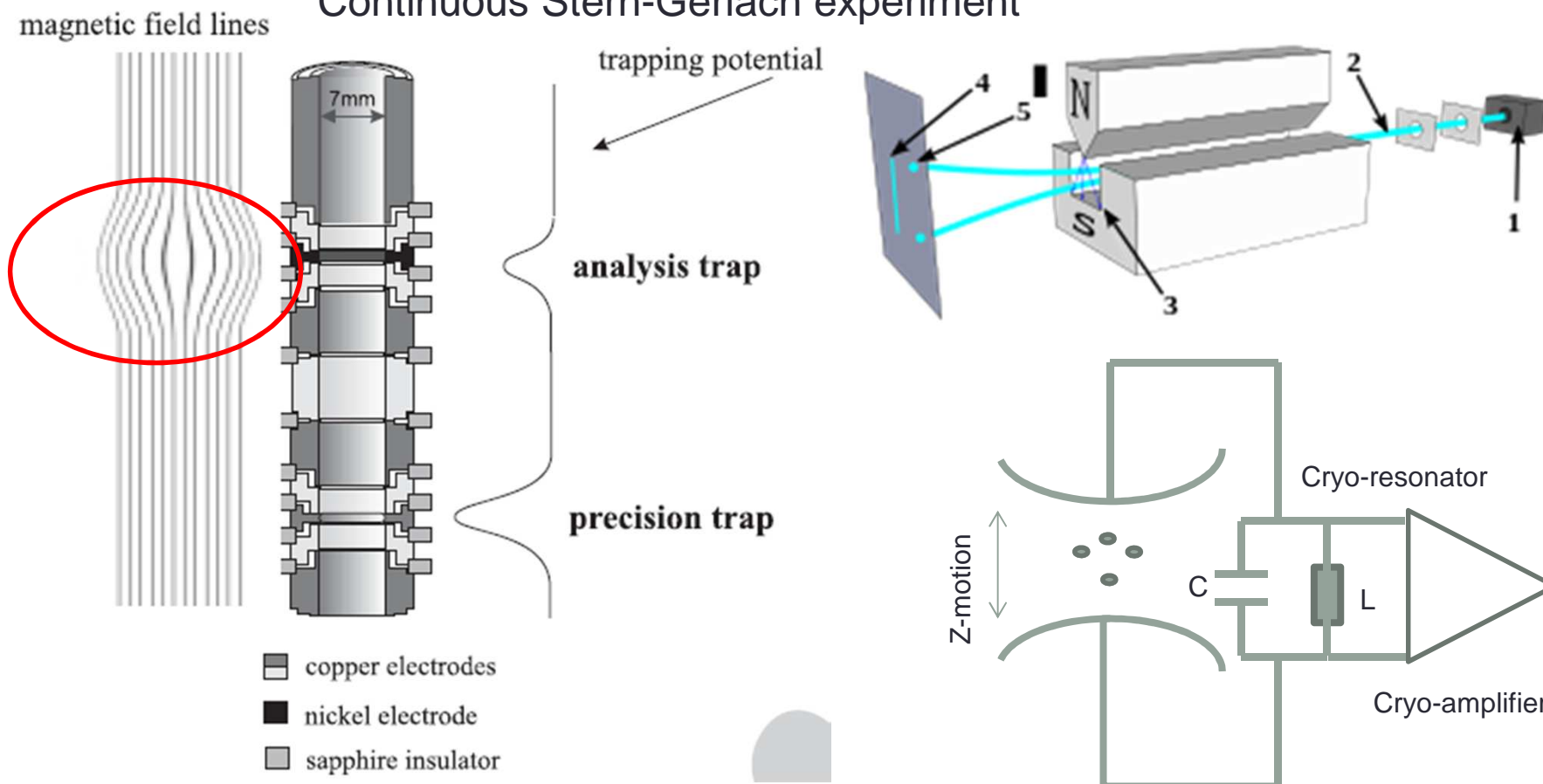
Reminder: Stern- Gerlach apparatus



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G-factor measurements: PT

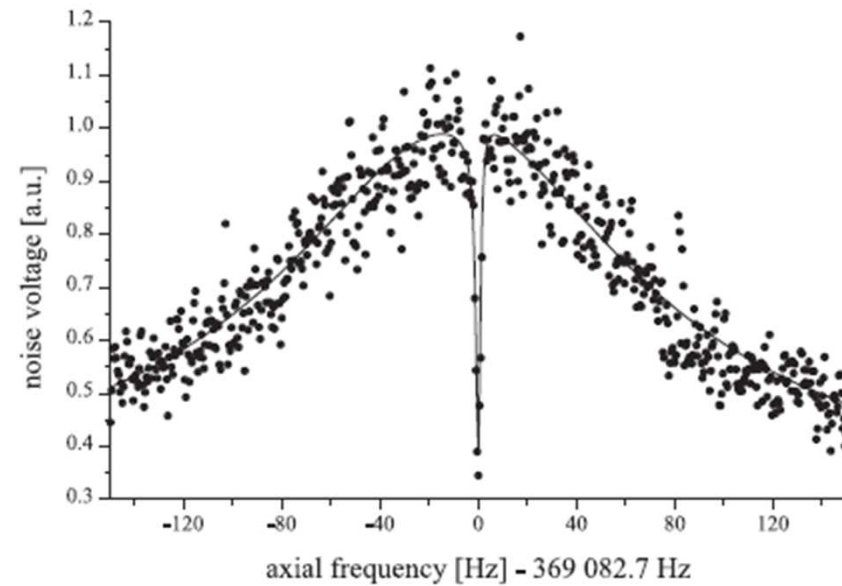
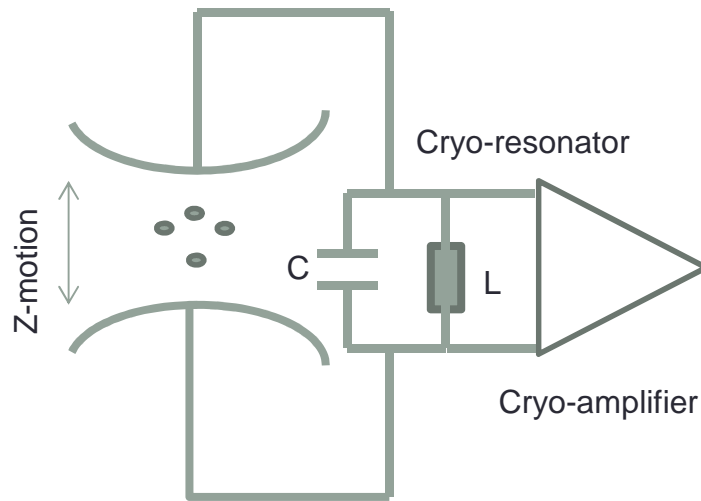
Continuous Stern-Gerlach experiment



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G-factor measurements: PT

Continuous Stern-Gerlach experiment

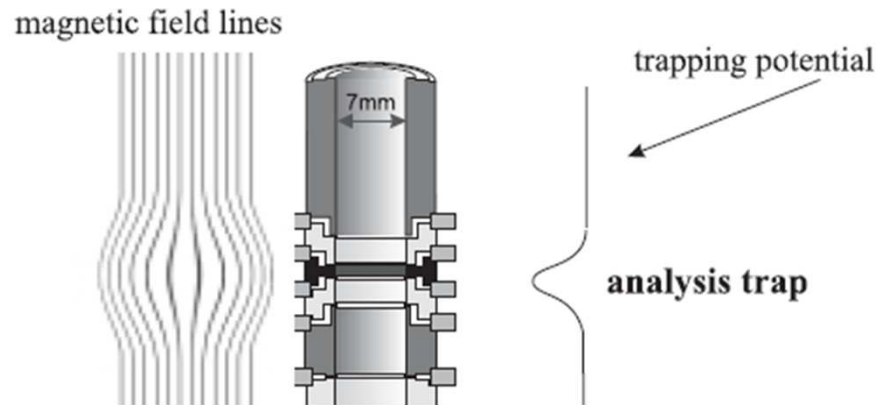


Center frequency can be determined to
mHz precision at a frequency of ~ 300 Hz

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G-factor measurements: PT

Continuous Stern-Gerlach experiment



$$F \hat{z} = -\nabla(\vec{\mu} \cdot \vec{B}) = -2\mu_z B_2 z$$

The z-motion remains a SHO with a modified frequency

$$\omega_z = \omega_{z0} + \frac{1}{2} \delta\omega_z = \omega_{z0} + \frac{\mu_z B_2}{M \omega_{z0}}$$

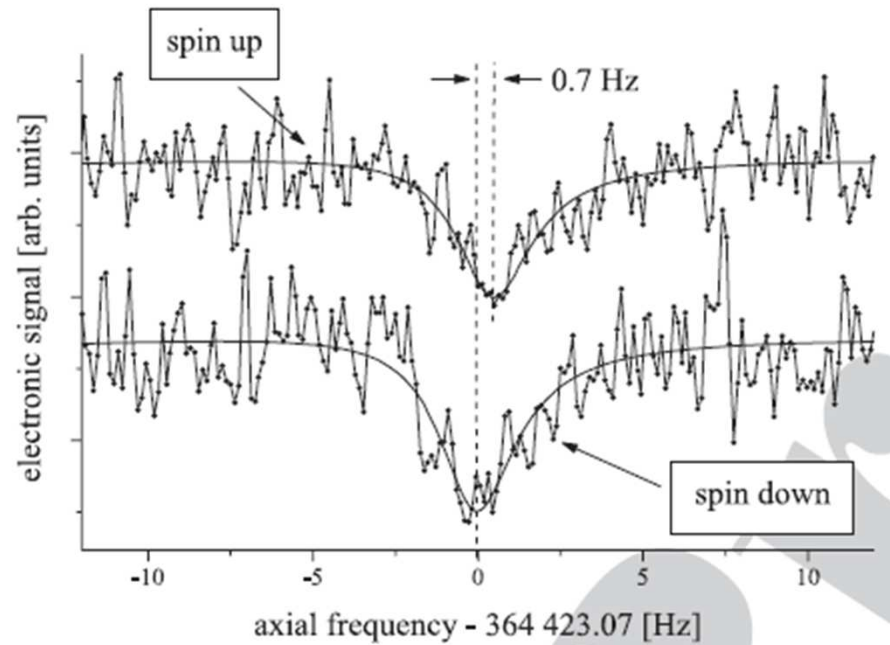
Can be calculated or measured $\sim 1\text{T} / \text{sqcm}$

$$\vec{B} = \vec{B}_0 + 2B_2 \left(\frac{z^2 - r^2}{2} \hat{z} - z\hat{r} \right)$$

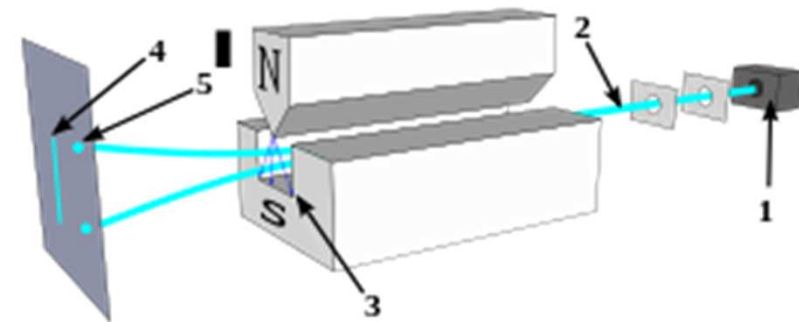
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G-factor measurements: PT

Continuous Stern-Gerlach experiment



$$\omega_z = \omega_{z0} + \frac{1}{2} \delta\omega_z = \omega_{z0} + \frac{\mu_z B_2}{M \omega_{z0}}$$

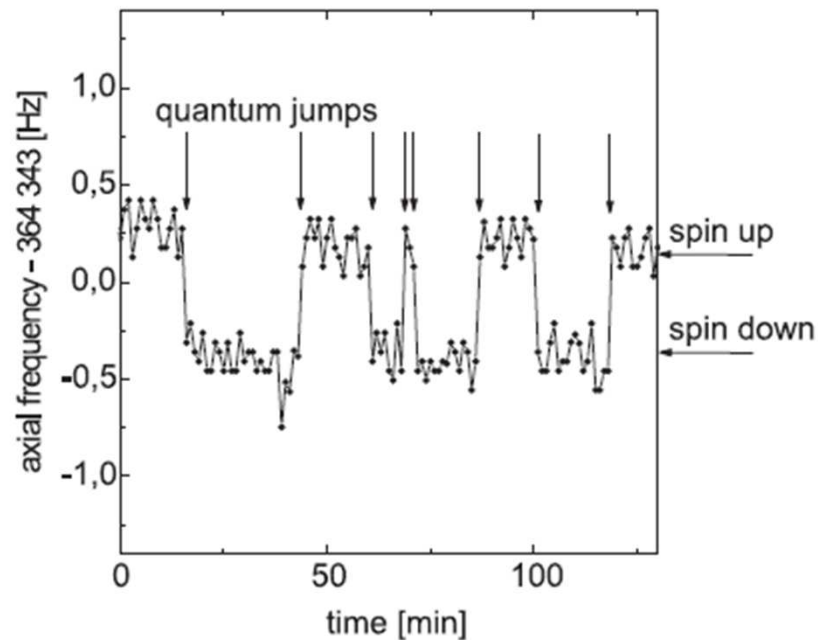


Acts as the screen

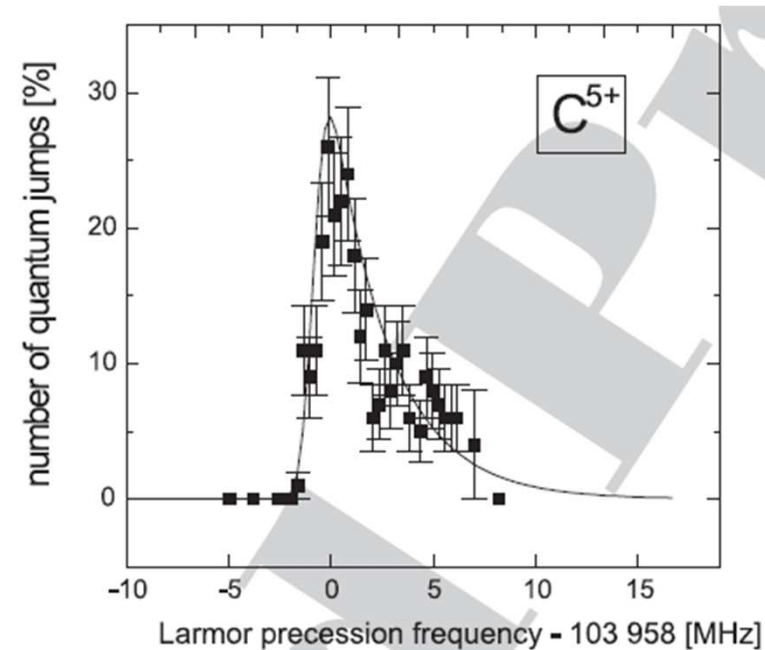
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G-factor measurements: PT

Continuous Stern-Gerlach experiment



Spin flip transitions



Determination of spin-flip frequency

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